Nuclear Structure of $^{122\text{-}134}$Xe Isotopes

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The potential energy surfaces, $V(\beta,\gamma)$, for a series of Xenon isotopes $^{122-134}$Xe have been calculated. The relatively flat potential to ^{130}Xe and energy ratio $E^+_{4_1}/E^+_{2_1} = 2.2$ show $E(5)$ symmetry to the nucleus which is laying in the transition region from γ-soft to vibrational characters. The interacting boson approximation model ($IBA-1$) has been used in calculating levels energy and electromagnetic transition probabilities $B(E2)'s$. Back bending is observed for $^{122-130}$Xe. The calculated values are compared to the available experimental data and show reasonable agreement.

1 Introduction

The chain of $^{122-134}$Xe isotopes is of great interest because of the existence of transitional nuclei where the nuclear structure changes from rotational to vibrational shapes. Many authors studied this area of isotopes experimentally and theoretically.

Experimentally, the mass of $^{122-134}$Xe isotopes [1] were detected on line using mass separator ISOLDE/CERN while the lifetimes of the low lying states in $^{122-134}$Xe were measured using Doppler-Shift [2] technique.

Theoretically, many authors studied this series of isotopes useing different theoretical models as algebric $sp(4)$ shell model [3], cranked Strutinsky method [4], relativistic mean field theory [5, 6], isospin-dependent lattice gas model [7, 8], general Bohr Hamiltonian [9], quadrupole-quadrupole plus pairing model [10], cranked Hartree-Fock-Bogoliubov model [11, 12] and interacting boson approximation model [13, 17]. They reported:

1. the reduced transition probabilities for Yrast spectra up to $I^+ = 10$;
2. the existance of shape transitions as well as $E(5)$ and $X(5)$ symmetry nuclei,
3. the occurrence of backbending in $^{122-130}$Xe nuclei, and
4. M1 transition probabilities between the mixed-symmetry and fully symmetric states.

2 Interacting Boson Approximation Model

The IBA-1 model [18] was applied to the positive parity low-lying states in even-even $^{122-134}$Xe isotopes. The proton, π, and neutron, ν, bosons are treated as one boson and the system is considered as an interaction between s-bosons and d-bosons. Creation ($s^\dagger d^\dagger$) and annihilation ($s\tilde{d}$) operators are for s and d bosons. The Hamiltonian employed for the present calculation is given as:

$$\begin{aligned} H \;=\; & EPS\cdot n_d + PAIR\cdot(P\cdot P)+ \\ & +\tfrac{1}{2}\,ELL\cdot(L\cdot L)+\tfrac{1}{2}\,QQ\cdot(Q\cdot Q)+ \\ & +5\,OCT\cdot(T_3\cdot T_3)+5HEX\cdot(T_4\cdot T_4), \end{aligned} \tag{1}$$

where

$$P\cdot p = \frac{1}{2}\left[\begin{array}{l}\left\{(s^\dagger s^\dagger)^{(0)}_0 - \sqrt{5}(d^\dagger d^\dagger)^{(0)}_0\right\}x \\ \left\{(ss)^{(0)}_0 - \sqrt{5}(\tilde{d}\tilde{d})^{(0)}_0\right\}\end{array}\right]^{(0)}_0, \tag{2}$$

$$L\cdot L = -10\sqrt{3}\left[(d^\dagger\tilde{d})^{(1)}x\,(d^\dagger\tilde{d})^{(1)}\right]^{(0)}_0, \tag{3}$$

$$Q\cdot Q = \sqrt{5}\left[\begin{array}{l}\left\{(S^\dagger\tilde{d}+d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger\tilde{d})^{(2)}\right\}x \\ \left\{(s^\dagger\tilde{d}++\tilde{d}s)^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger\tilde{d})^{(2)}\right\}\end{array}\right]^{(0)}_0, \tag{4}$$

$$T_3\cdot T_3 = -\sqrt{7}\left[(d^\dagger\tilde{d})^{(2)}x\,(d^\dagger\tilde{d})^{(2)}\right]^{(0)}_0, \tag{5}$$

$$T_4\cdot T_4 = 3\left[(d^\dagger\tilde{d})^{(4)}x\,(d^\dagger\tilde{d})^{(4)}\right]^{(0)}_0. \tag{6}$$

In the previous formulas, n_d is the number of bosons; $P\cdot P$, $L\cdot L$, $Q\cdot Q$, $T_3\cdot T_3$ and $T_4\cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and $PAIR$, ELL, QQ, OCT, HEX are the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

3 Results and discussion

3.1 The potential energy surfaces, (PESs)

The PESs [19], $V(\beta,\ \gamma)$, for Xenon isotopes as a function of the deformation parameters β and γ have been calculated using :

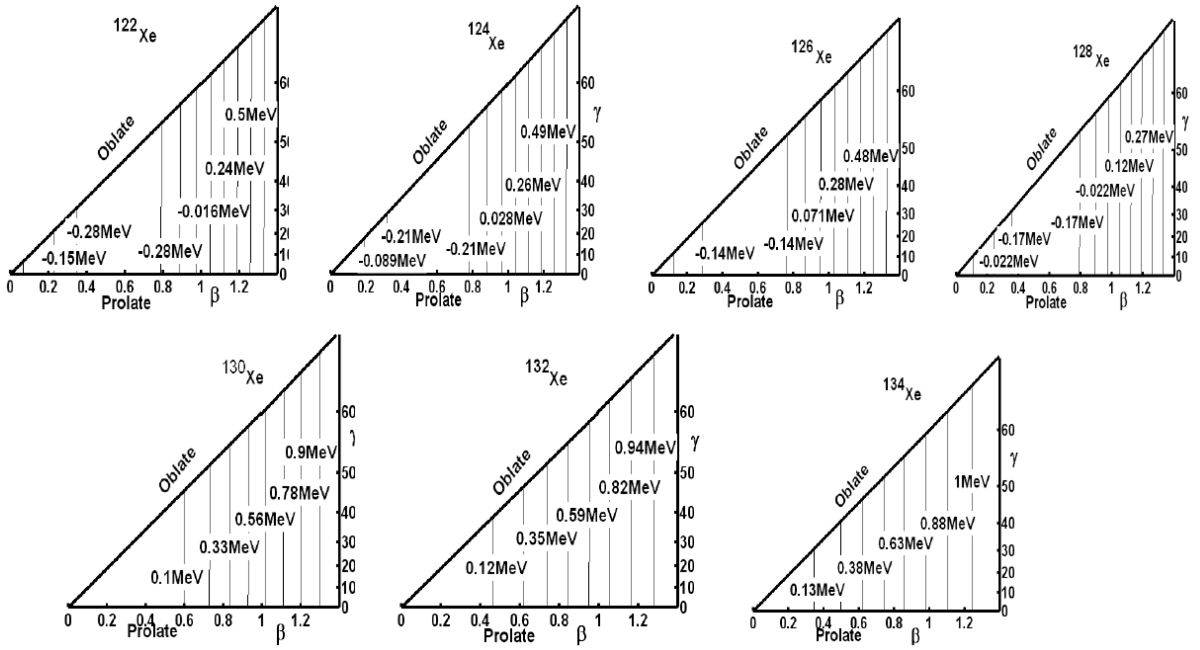

Fig. 1: Contour plot of the potential energy surfaces for $^{122-134}$Xe nuclei.

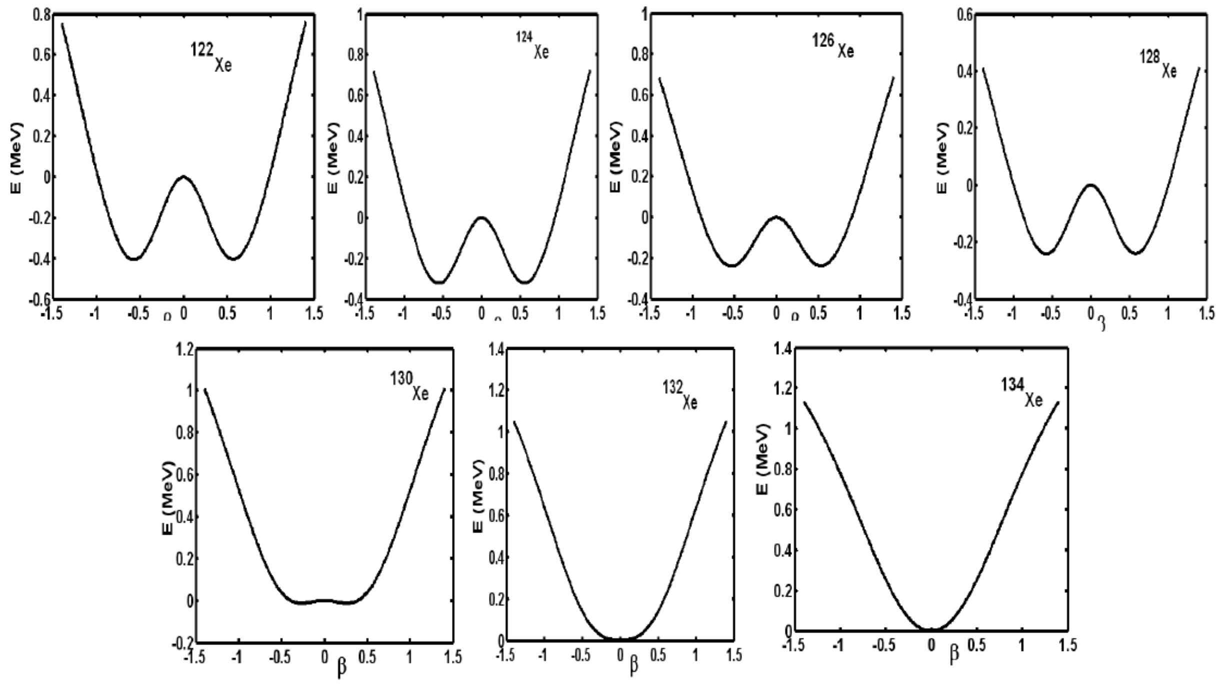

Fig. 2: Potential energy surfaces for $^{122-134}$Xe nuclei at $\gamma = 0^o$ (Prolate) and $\gamma = 60^o$ (Oblate).

nucleus	EPS	$PAIR$	ELL	QQ	OCT	HEX	$E2SD(eb)$	$E2DD(eb)$
^{122}Xe	0.4700	0.0000	0.0216	−0.0200	0.0000	0.00000	0.1390	−0.4112
^{124}Xe	0.4680	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1280	−0.3786
^{126}Xe	0.4490	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1260	−0.3727
^{128}Xe	0.4720	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1410	−0.4171
^{130}Xe	0.5420	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1500	−0.4437
^{132}Xe	0.6450	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1460	−0.4319
^{134}Xe	0.8020	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1480	−0.4378

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

I_i^+ I_f^+	^{122}Xe	^{124}Xe	^{126}Xe	^{128}Xe	^{130}Xe	^{132}Xe	^{134}Xe
$0_1{}^*Exp.\ 2_1$	1.40(6)	0.96(6)	0.770(25)	0.750(40)	0.65(5)	0.460(30)	0.34(6)
0_1 Theo. 2_1	1.4038	0.9651	0.7691	0.7575	0.6575	0.4684	0.3451
2_1 0_1	0.2808	0.1930	0.1538	0.1515	0.1315	0.0937	0.0690
2_2 0_1	0.0057	0.0033	0.0022	0.0015	0.0007	0.0002	0.0001
2_2 0_2	0.1552	0.0979	0.0741	0.0684	0.0567	0.0412	0.0343
2_3 0_1	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
2_3 0_2	0.1640	0.1278	0.1047	0.1077	0.0926	0.0583	0.0298
2_3 0_3	0.0465	0.0248	0.0161	0.0133	0.0113	0.0091	0.0086
2_4 0_3	0.0766	0.0355	0.0198	0.0121	0.0064	0.0025	——
2_4 0_4	0.1031	0.0886	0.0784	0.0867	0.0839	0.0683	——
4_1 2_1	0.5297	0.3583	0.2787	0.2650	0.2186	0.1447	0.0941
4_1 2_2	0.0487	0.0316	0.0239	0.0227	0.0194	0.0145	0.0124
4_1 2_3	0.0737	0.0562	0.0452	0.0456	0.0386	0.0240	0.0122
6_1 4_1	0.6735	0.4529	0.3448	0.3183	0.2482	0.1465	0.0714
6_1 4_2	0.0476	0.0326	0.0254	0.0259	0.0244	0.0198	0.0182
6_1 4_3	0.0563	0.0428	0.0337	0.0332	0.0261	0.0127	——
8_1 6_1	0.7369	0.4875	0.3586	0.3139	0.2199	0.0979	——
8_1 6_2	0.0409	0.0290	0.0230	0.0246	0.0248	0.0214	——
8_1 6_3	0.0438	0.0319	0.0237	0.0210	0.0127	——	——
10_1 8_1	0.7363	0.4717	0.3269	0.2567	0.1362	——	——
10_1 8_2	0.0347	0.0252	0.0202	0.0223	0.0237	——	——

Table 2: Theoretically calculated reduced transition probabilities, $B(E2)'s$ in $e^2\,b^2$. *Ref. [27]

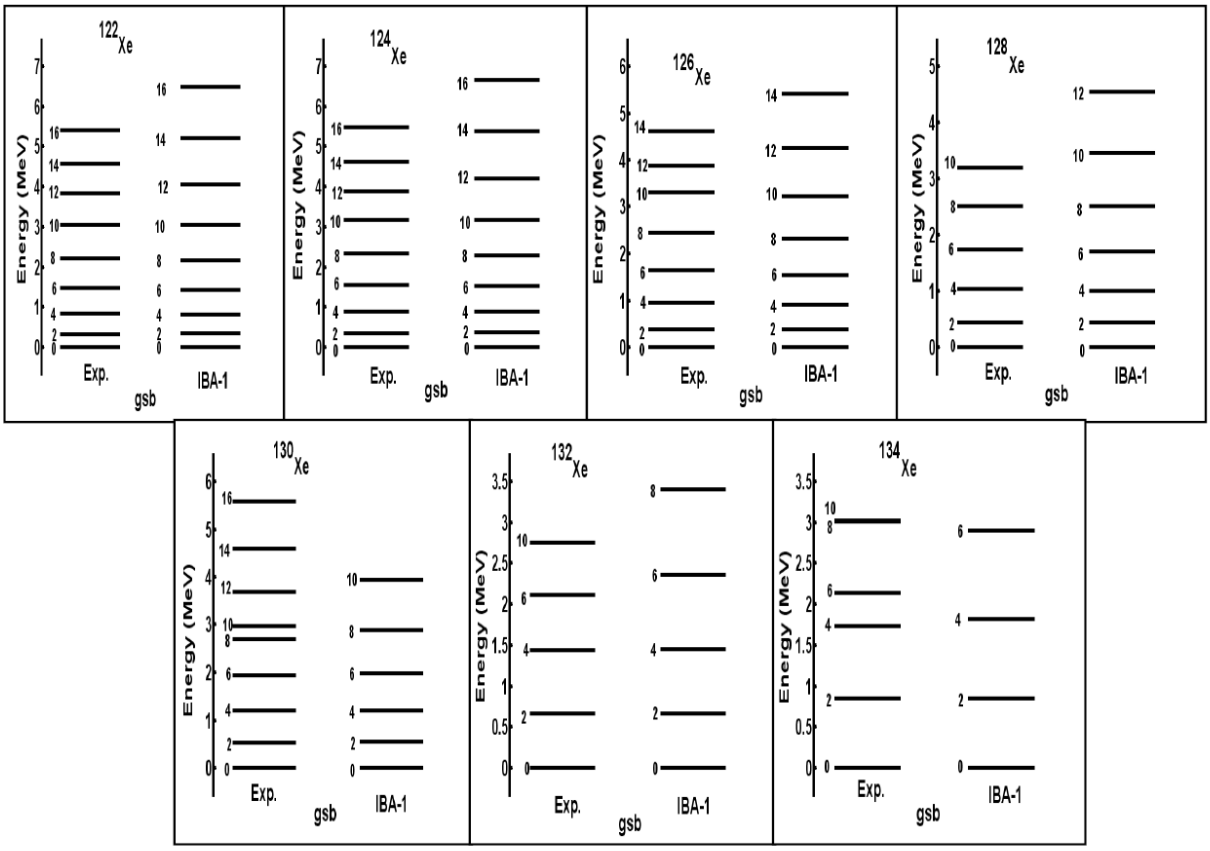

Fig. 3: Comparison between experimental [20–26] and theoretical (IBA) energy levels.

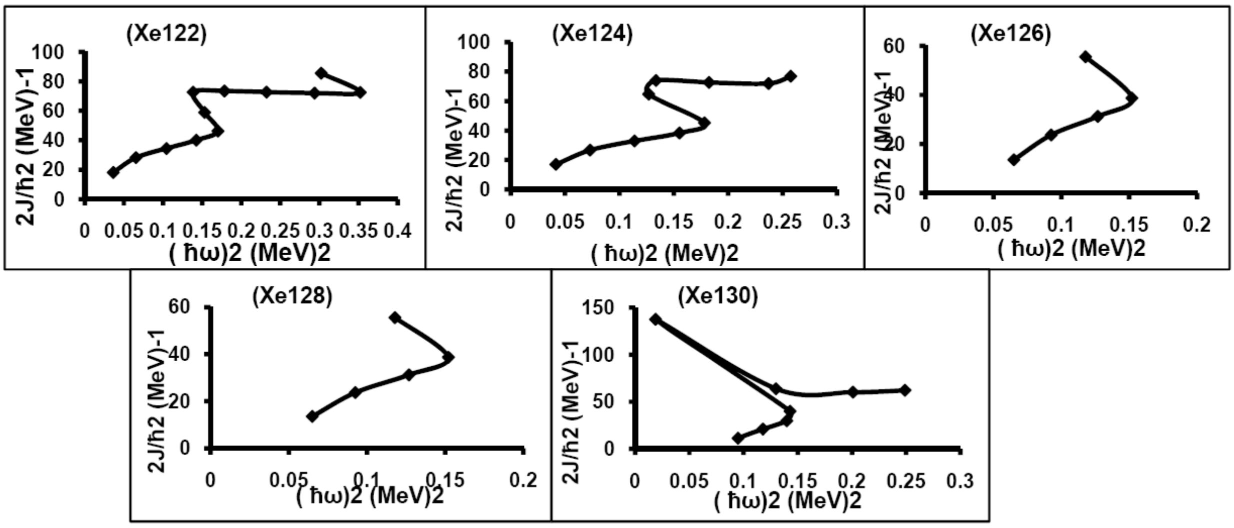

Fig. 4: Back bending in $^{122-134}$Xe isotopes.

$$E_{N_\Pi N_\nu}(\beta,\gamma) = <N_\pi N_\nu;\beta\gamma|H_{\pi\nu}|N_\pi N_\nu;\beta\gamma> =$$
$$= \zeta_d(N_\nu N_\pi)\beta^2(1+\beta^2) + \beta^2(1+\beta^2)^{-2}\times$$
$$\times\left\{kN_\nu N_\pi[4-(\bar{X}_\pi\bar{X}_\nu)\beta\cos 3\gamma]\right\}+ \quad (7)$$
$$+\left\{[\bar{X}_\pi\bar{X}_\nu\beta^2] + N_\nu(N_\nu-1)\left(\frac{1}{10}c_0+\frac{1}{7}c_2\right)\beta^2\right\},$$

where

$$\bar{X}_\rho = \left(\frac{2}{7}\right)^{0.5} X_\rho, \quad \rho = \pi\ or \nu\,. \quad (8)$$

The calculated PESs, $V(\beta,\gamma)$, for Xenon series of isotopes are presented in Fig. 1 and Fig. 2. They show that $^{122-128}$Xe nuclei are deformed and the two wells on both oblate and prolate sides are nearly equals and O(6) characters is expected to these nuclei. ^{130}Xe has flat potential energy, Fig. 2, which indicates that the nucleus is $E(5)$ symmetry and confirmed by the energy ratio $R = E^+_{4_1}/E^+_{2_1} = 2.2$ as well as it is laying also in the transition from γ- unstable, $O(6)$, to vibrational, $U(5)$, nuclei while, $^{132,134}Xe$ are vibrational like nuclei.

3.2 Energy spectra and transition rates

IBA-1 model has been used in calculating the energy of the positive parity low-lying levels of Xenon series of isotopes. Comparison between the experimental spectra [20–26] and our calculations, using values of the model parameters given in Table 1, are illustrated in Fig. 3. The agreement between the low-laying calculated energy levels and their corresponding experimental values is fairly good but for higher states theoretical values are slightly higher. We believe that is due to the change of the projection of the angular momentum which may be due to band crossing and change in angular momentum.

The electric quadrupole transition operator [18] employed in this study is given by:

$$T^{(E2)} = E2SD\cdot(s^\dagger\tilde{d}+d^\dagger s)^{(2)} + \frac{1}{\sqrt{5}}E2DD\cdot(d^\dagger\tilde{d})^{(2)}\,. \quad (9)$$

The reduced electric quadrupole transition rates between $I_i \to I_f$ states are given by

$$B(E_2, I_i - I_f) = \frac{[< I_f \parallel T^{(E_2)} \parallel I_i >]^2}{2I_i+1}\,. \quad (10)$$

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E2)$ for these series of nuclei. The only measured $B(E2, 0^+_1 \to 2^+_1)$'s are presented, in Table 2 for comparison to the calculated values. The parameters $E2SD$ and $E2DD$, displayed in Table 1, are used in the present calculation of the transition rates $B(E2)'s$ and then normalized to the experimentally known ones [27]. In our calculations we did not introduce any new parameters.

3.3 Back bending

The moment of inertia J and energy parameters $\hbar\omega$ are calculated [28]using equations (11, 12):

$$\frac{2J}{\hbar^2} = \frac{4I-2}{\Delta E(I\to I-2)}\,, \quad (11)$$

$$(\hbar\omega)^2 = (I^2-I+1)\left[\frac{\Delta E(I\to I-2)}{(2I-1)}\right]^2. \quad (12)$$

The plots in Fig. 4 show back bending for $^{122-126}$Xe at $I^+ = 10$ while at $I^+ = 12$ for ^{128,130}Xe and this is in agreement with the work done by other authors [29]. Back bending in Xenon isotopes in higher states is explained [10] as due to partial rotational alignment of a pair of neutrons in the $1h_{1/2}$ neutron orbit near the Fermi surface.

4 Conclusions

The IBA-1 model has been applied successfully to $^{122-134}$Xe isotopes and we have got:

1. The ground state bands are successfully reproduced;
2. The potential energy surfaces are calculated and show $O(6)$ characters to $^{122-128}$Xe isotopes where the prolate and oblate depths are equal;
3. Flat potential energy to ^{130}Xe and energy ratios confirmed that the nucleus is an $E(5)$ symmetry;
4. ^{132,134}Xe nuclei show vibrational-like characters;
5. Electromagnetic transition rates, $B(E2)'s$, are calculated, then normalized to experimental B(E2, 0_1 – 2_1) values and then compared to the available data, and
6. Back bending for $^{122-126}$Xe have been observed at angular momentum $I^+ = 10$ and at $I^+ = 12$ for ^{128,130}Xe.

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