# Key to the Mystery of Dark Energy: Corrected Relationship between Luminosity Distance and Redshift

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A new possible explanation to the luminosity distance ($D_L$) and redshift ($Z$) measurements of type Ia supernovae (SNeIa) is developed. Instead of modifying the theory of general relativity or the Friedmann equation of cosmology with an extra scalar field or unknown energy component (e.g., dark energy), we re-examine the relationship between the luminosity distance and the cosmological redshift ($D_L - Z$). It is found that the $D_L - Z$ relation previously applied to connect the cosmological model with the measured SNeIa data is only valid for nearby objects with $Z \ll 1$. The luminosity distances of all distant SNela with $Z \gtrsim 1$ had been underestimated. The newly derived $D_L - Z$ relation has an extra factor $\sqrt{1+Z}$, with which the cosmological model exactly explains all the SNeIa measurements without dark energy. This result indicates that our universe has not accelerated and does not need dark energy at all.

## 1 Introduction

There are five possible ways to explain the luminosity distance ($D_L$) and redshift ($Z$) measurements of type Ia supernovae (SNeIa) according to the general relativity (GR), which derives the Friedmann equation (FE) with the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric of the 4D spacetime (Figure 1).

The most simple and direct way is the famous Lambda Cold Dark Matter (ΛCDM) model, currently accepted as the standard one, which introduces a cosmological constant Λ to the field equation of GR (Eq. 1), referred as a candidate of dark energy [1-2],

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the Einsteinian curvature tensor of spacetime, $T_{\mu\nu}$ is the energy-momentum tensor of matter, $c$ is the light speed in free space, and $G$ is the gravitational constant. The cosmological constant Λ was first introduced actually by Albert Einstein himself into his field equation, Eq. (1), in order to have a static universe about a century ago, and then discarded after the universe was found to be expanding [3].

The second way that has also been comprehensively studied is the scalar-tensor (S-T) theory, which introduces a scalar field Φ, usually time-dependent, to the action of spacetime ($S_G$) [4-5]. This category includes also the four-dimensional $f(R)$, galileon, and five-dimensional Kaluza-Klein theories with scalar fields [6-12]. The third way is the scalar perturbation (SP) theory, which inputs perturbation scalars Ψ and Φ, usually time-independent, into the FLRW metric rather than into the action $S_G$ [13-15]. The S-T and SP theories may be equivalent because both attempt to modify the curvature of spacetime. The cosmological constant Λ can also be added to $S_G$ for a less curvature of spactime or to the action of matter $S_M$ for an extra energy component. The fourth possible way is according the black hole universe (BHU) model, recently developed by the author [16-18], in which the expansion and acceleration of the universe are driven by the external energy.

The procedures that the above four models commonly follow in the explanation of the SNeIa measurements include the following four steps: (1) Modifying the FE with an appropriate input of Λ, scalar field, perturbation, or external energy; (2) Determining the expansion rates (Hubble parameter) of the universe according to their modified FEs; (3) Submitting their expansion rates into the $D_L - Z$ relation; (4) Comparing the obtained redshift dependence of their luminosity distances with the SNeIa measurements. Fitting the models to the data determines the amount of the input such as $\Omega_\Lambda \sim 0.73$ for the ΛCDM model [1-2] and $\ddot{M}(t) \sim 10^{17}$ kg/s$^2$ for the BHU model [18].

In this paper, a new and most probable explanation for the SNela measurements is developed without attempting to modify the theory of gravitation or the model of cosmology by inserting one or more fields or constants into GR or FE. Instead, we will re-examine the $D_L - Z$ relationship that connects the cosmological model with the SNela data. We will derive a new $D_L - Z$ relation and further compare this new relation with the SNela measurements to examine whether or not our universe needs the dark energy or has recently accelerated.

## 2 Mystery of Dark Energy

The greatest unsolved problem in the modern cosmology is the mystery of dark energy [19]. This currently most accepted hypothesis for the standard cosmological model to quantitatively explain the measurements of distant type Ia supernovae strongly relies on the $D_L - Z$ relation that is used to bridge the measured SNeIa data and the theoretical model of cosmology.

However, the $D_L - Z$ relation that was usually applied to analyze the measurements of distant type-Ia supernovae,

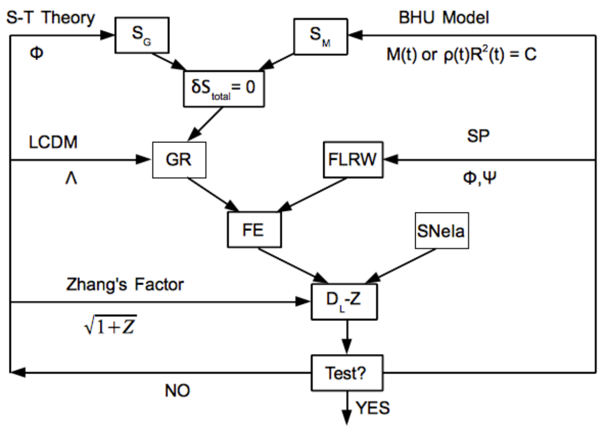


Fig. 1: Flow chat for five possible ways to explain the luminosity distance and redshift measurements of type Ia supernovae. They are: (1) GR with the cosmological constant $\Lambda$; (2) Gravitational theory with a scalar field $\Phi$; (3) FLRW metric with perturbations $\Phi$ and $\Psi$; (4) Black hole universe model with increasing input of external energy $\ddot{M} > 0$; and (5) Luminosity distance-redshift relation with a factor of $\sqrt{1+Z}$. This Study focuses on the fifth possible explanation.

$$D_{\rm L} \simeq c(1+Z)R(t_o)\int_{t_e}^{t_o}\frac{dt}{R(t)}, \tag{2}$$

is an approximate expression that is only valid for nearby objects with $Z \ll 1$ in a flat universe [20]. Here $t_e$ is the time when the light is emitted, $t_o$ is the time when the light is observed, $R(t)$ is the scale factor, which is defined from the FLRW metric [21-24],

$$ds^2 = -c^2dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{3}$$

and governed by the Friedmann equation [25],

$$H^2(t) \equiv \frac{\dot{R}^2(t)}{R^2(t)} = \frac{8\pi G\rho_M(t)}{3} - \frac{kc^2}{R^2(t)} + \frac{\Lambda}{3}, \tag{4}$$

according to the standard cosmological model, where $\rho_M(t)$ is the matter density, $k$ is the curvature ($k = 0$ for a flat universe), $\Lambda$ is the cosmological constant (or a candidate of dark energy), the coordinates $\{t, r, \theta, \phi\}$ are co-moving coordinates, and $H(t)$ is the Hubble parameter, which, at the present time, is called the Hubble constant and measured at $H_0 \sim 70$ km/s/Mpc [3, 26-27].

In the FLRW universe due to the time dependent scalar factor, light gets redshifted. According to the theory of GR, light travels on null geodesics (i.e., $ds^2 = 0$). Then along a radial light path, we have

$$\frac{cdt}{R(t)} = \frac{dr}{\sqrt{1-kr^2}}. \tag{5}$$

It follows from Eq. (5) that

$$\int_{t_e}^{t_o}\frac{cdt}{R(t)} = \int_{t_e+\delta t_e}^{t_o+\delta t_o}\frac{cdt}{R(t)} = \int_{r_1}^{0}\frac{dr}{\sqrt{1-kr^2}}. \tag{6}$$

Subtracting the first integral from the second and assuming $\delta t_e, \delta t_o << R(t)/\dot{R}(t)$, we get

$$\frac{\delta t_e}{R(t_e)} = \frac{\delta t_o}{R(t_o)}. \tag{7}$$

Since $\delta t_e = 1/\nu_e = \lambda_e/c$ and $\delta t_o = 1/\nu_o = \lambda_o/c$, the cosmological redshift $Z$ can be determined according to the scale factor $R(t)$ as

$$1+Z \equiv \frac{\lambda_o}{\lambda_e} = \frac{\nu_e}{\nu_o} = \frac{\delta t_o}{\delta t_e} = \frac{R(t_o)}{R(t_e)}. \tag{8}$$

Here $\lambda$ and $\nu$ are the light wavelength and frequency, respectively. Light from a source object is redshifted because the time interval or scale factor is increased. The reason for an individual photon to be observed with smaller frequency (or energy) is due to that the time interval of observation is greater.

The scale factor is related to the energy and curvature via Eq. (4) and to the redshift via Eq. (8). In terms of Eqs. (4) and (8), the luminosity distance-redshift relation Eq. (2) can be reformed as

$$\begin{aligned} D_{\rm L} &\simeq c(1+Z)\int_0^Z\frac{dz'}{H(z')} \\ &= \frac{c}{H_0}(1+Z)\int_0^Z\frac{dz'}{\sqrt{\Omega_M(1+z')^3+\Omega_\Lambda}}, \end{aligned} \tag{9}$$

with $1 = \Omega_M + \Omega_\Lambda$. For an arbitrary $k$, Eq. (9) is generally represented as

$$\begin{aligned} D_{\rm L} \simeq\ & \frac{c}{H_0\sqrt{|\Omega_k|}}(1+Z)S\left(\sqrt{|\Omega_k|}\ \times\right. \\ & \left.\int_0^Z\frac{dz'}{\sqrt{\Omega_M(1+z')^3+\Omega_k(1+z')^2+\Omega_\Lambda}}\right), \end{aligned} \tag{10}$$

where

$$S(x) = \begin{cases} \sin(x), & \text{if } k<0 \\ x, & \text{if } k=0 \\ \sinh(x), & \text{if } k>0 \end{cases} \tag{11}$$

and $1 = \Omega_M + \Omega_\Lambda + \Omega_k$.

Comparing the luminosity distance and redshift measurements of distant SNeIa with the luminosity distance-redshift relation determined in terms of Eqs. (2), (4), and (8) or Eq. (9) or Eq. (10) with $k = 0$, two supernova research groups [1-2], respectively, claimed that the universe has recently accelerated, so that the universe is dominated ($\Omega_\Lambda \sim 0.73$) by the dark energy.

However, re-examining the derivation of the luminosity distance-redshift relation, Eq. (2) so that Eqs. (9) and (10),

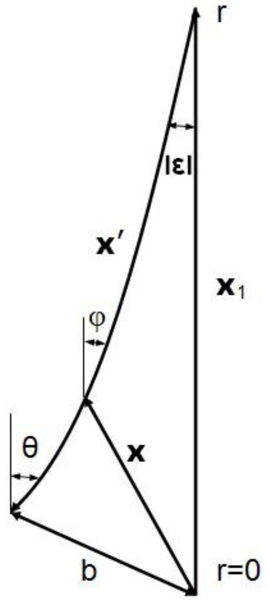


Fig. 2: Quantities used in the calculation of parallaxes and apparent luminosities [20]. The angles and the curvature of the light ray are greatly exaggerated.

we find that this relation is just an approximate relation only valid for nearby objects with $Z \ll 1$. Certainly, we cannot use it to correctly figure out the measurements of distant type Ia supernovae with $Z \gtrsim 1$. In the following, we will derive a new, more accurate and applicable also to distant objects, luminosity distance-redshift relation, which is perfectly consistent with all the measurements of type Ia supernovae without the input dark energy.

## 3 New $D_L - Z$ Relation

Now, the luminosity distance-redshift relation is derived by following the standard method as shown by [20] that calculates the parallaxes and apparent luminosities according to the path of light rays that leave from a source at $t = t_e$ and $r = r_e = R(t_e)r_1$ and pass to the observer at $t = t_o$ near $r = 0$ (see Figure 2). At the observation time $t = t_o$, the light source locates at $r = r_o = R(t_o)r_1$. Here, $r_1$ is the commoving distance defined by

$$r_1 = c\int_{t_e}^{t_o} \frac{dt}{R(t)}, \tag{12}$$

from the FLRW metric.

In the coordinate system $x'^{\mu}$ in which the light source is at the origin, the ray path is given by a position vector

$$\vec{x}' = \vec{n}\rho, \tag{13}$$

where $\vec{n}$ is a fixed unit vector and $\rho$ is a variable positive parameter describing positions along the path. The coordinate system $x'^{\mu}$ can be transformed to another coordinate system $x^{\mu}$ in which the observer is at the origin (e.g., the center of the telescope) and the light source is at $\vec{x}_1$. In the observer coordinate system, the ray path can be represented by (Eq. 14.4.2 of [20])

$$\vec{x} = \vec{x}' + \vec{x}_1\left(\left(1-kx'^2\right)^{1/2} - \left\{1-\left(1-kx_1^2\right)^{1/2}\right\}\frac{(\vec{x}'\cdot\vec{x}_1)}{x_1^2}\right). \tag{14}$$

For a flat universe ($k = 0$), the ray path in the coordinate system (Eq. 14) can be simplified as

$$\vec{x} = \vec{x}' + \vec{x}_1. \tag{15}$$

The parametric equation of the ray path, given by substituting Eq. (13) in Eq. (15), is then

$$\vec{x}(\rho) = \vec{n}\rho + \vec{x}_1. \tag{16}$$

The distance of light ray to the origin in the observer coordinate system will be

$$\begin{aligned} |\vec{x}| &= \sqrt{\left(\vec{x}' + \vec{x}_1\right)\cdot\left(\vec{x}' + \vec{x}_1\right)} \\ &= \sqrt{x_1^2 + \rho^2 - 2x_1\rho\cos\phi} \\ &\sim \sqrt{(x_1-\rho)^2 + x_1\rho\phi^2}, \end{aligned} \tag{17}$$

where we have considered the angle $\phi$ between $\vec{n}$ and $-\vec{x}_1$ is small and thus $\cos\phi \sim 1 - \phi^2/2$.

At the emission time $t_e$, we have

$$\rho|_{t=t_e} = 0, \tag{18}$$

$$|\vec{x}|_{t=t_e} = |\vec{x}_1|_{t=t_e} = r_e = r_1R(t_e), \tag{19}$$

$$\phi|_{t=t_e} = |\vec{\epsilon}|, \tag{20}$$

while at the observation time $t_o$, we have

$$\rho|_{t=t_0} = r_o = r_1R(t_o), \tag{21}$$

$$|\vec{x}|_{t=t_o} = b, \tag{22}$$

$$|\vec{x}_1|_{t=t_o} = r_o = r_1R(t_o), \tag{23}$$

$$\phi|_{t=t_o} = \theta = |\vec{\epsilon}|R(t_o)/R(t_e). \tag{24}$$

Substituting the quantity properties Eqs. (21)-(24) at $t = t_o$ into Eq. (17), we obtain the impact parameter as

$$b = R(t_o)r_1\theta = \frac{R^2(t_o)}{R(t_e)}r_1|\vec{\epsilon}|. \tag{25}$$

To calculate apparent luminosities, we consider a circular telescope mirror of radius $b$, placed with its center at the origin and its normal along the line of sight to the light source.

The fraction of all emitted photons that reach the mirror is the ratio of the solid angle to $4\pi$,

$$\frac{\pi|\vec{\epsilon}|^2}{4\pi} = \frac{\pi b^2}{4\pi r_1^2}\frac{R^2(t_e)}{R^4(t_o)}. \quad (26)$$

Since light is red-shifted, the energy or frequency of each photon observed is reduced in comparison with the photon emitted by a factor of $R(t_e)/R(t_o)$. This energy or frequency reduction is equivalent to the increase of the time interval for observation relative to that for emission. If the effect of the redshift on the apparent luminosity is considered, then we should not consider the effect of the time interval increase on the apparent luminosity. This is also consistent with the electromagnetic wave theory of light, from which the energy emitted per unit time of emission is only one redshift factor greater than the energy observed per unit time of observation. Therefore, the total power $P$ received by the mirror is the total power emitted by the source, its absolute luminosity $L$, times a factor $R(t_e)/R(t_o)$, and times the fraction (Eq. 26):

$$P = L\frac{\pi b^2}{4\pi r_1^2}\frac{R^3(t_e)}{R^5(t_o)}. \quad (27)$$

The apparent luminosity $l$ is the power per unit mirror area

$$l = \frac{P}{\pi b^2} = \frac{L}{4\pi r_1^2}\frac{R^3(t_e)}{R^5(t_o)}. \quad (28)$$

Then the luminosity distance can be obtained

$$D_{\rm L} = \left(\frac{L}{4\pi l}\right)^{1/2} = r_1 R(t_o)\left[\frac{R(t_o)}{R(t_e)}\right]^{3/2}$$

$$= c(1+Z)^{3/2}R(t_o)\int_{t_e}^{t_o}\frac{dt}{R(t)}. \quad (29)$$

The luminosity distance Eq. (29) derived here is $\sqrt{1+Z}$ times that we conventionally used, Eq. (2). This factor leads to an explanation of type Ia supernova measurements without dark energy. Using Eqs. (4) and (8) for a flat universe ($k = 0$) without dark energy ($\Lambda = 0$), we can integrate Eq. (29) and obtain the luminosity distance-redshift relation as

$$D_{\rm L} = \frac{2c}{H_0}(1+Z)\left(\sqrt{1+Z}-1\right). \quad (30)$$

Eq. (30) dees not include any free parameter and reduces to the Hubble law at $Z \ll 1$.

The two significant corrections, which have been made in the above derivation of the luminosity distance in comparison with the derivation done in [20] are: 1) $\theta$ is not about equal to $|\vec{\epsilon}|$ for a distant light source but increased by a factor $R(t_o)/R(t_e)$, and 2) the light is red-shifted and the time interval increases are equivalent in physics $\nu_o/\nu_e = \delta t_e/\delta t_o = R(t_e)/R(t_o)$ and thus they reduce the apparent luminosity only by $R(t_e)/R(t_o)$ rather than its square. This is also supported by the electromagnetic wave theory of light.

The early derivation, including the simplified version as given in [28] and other cosmological books, the fraction of the light received in a telescope of aperture $\pi b^2$ on earth is $\pi b^2/[4\pi r_1^2 R^2(t_o)]$ and so the factor $1/d^2$ in the formula for the apparent luminosity $l$ was replaced by $1/[r_1^2 R^2(t_o)]$. This replacement or modification for the apparent luminosity $l$ was made according to the view of the emitter rather than from the view of the observer. From the view of the emitter (or a person standing on the source object), all light rays radially diverge from the source object isotropically and in straight lines. All the photons emitted at $t_e$ reach the surface of the sphere drawn around the source object by radius $r_1 R(t_o)$. The angle of emission of a photon from the source object is equal to the angle of incidence of the photon to the mirror of telescope.

From the view of the observer, however, the source object is moving away in an increasing speed. The light rays travel in curved lines and anisotropically as shown in Figure 2. The angle of emission of a photon from the source object $|\vec{\epsilon}|$ is smaller than the angle of incidence of the photon to the mirror of telescope $\theta$ by a factor of $R(t_e)/R(t_o)$. That is, from the view of the observer, the factor $1/d^2$ in the formula for $l$ must be replaced with $1/[r_1^2 R^4(t_o)/R^2(t_e)]$ as shown in Eq. (26). On the other hand, according the electromagnetic wave theory, the energy of radiation does not depend on the frequency. Only the increase of time interval would reduce the apparent luminosity. This may be examinable in experiments using a sound wave.

Figure 3 plots the luminosity distance-redshift relation (red line) along with the type Ia supernova measurements (blue dots. Credit: Union 2.1 compilation of 580 SNIA data from Supernova Cosmology Project). In this plot the Hubble constant is chosen to be $H_0 \sim 70$ km/s/Mpc. In the upper panel of Figure 3, the distance modulus, which is defined by $\mu = 5\log_{10} D_{\rm L} - 5$ with $D_{\rm L}$ in parsecs, is plotted as a function of redshift; while in the lower panel of Figure 3, the distance modulus difference between the measured SNeIa data and analytical results derived from Eq. (30). The chi-square statistic is obtained as

$$\chi^2 = \sum_{j=1}^{580}\frac{\left(\mu_j^{\rm obs} - \mu_j^{\rm the}\right)^2}{\sigma_j^2} \sim 589. \quad (31)$$

Then the reduced chi-square is given by $\chi^2_{\rm red} = 589/580 \sim 1.015$. It is seen that the derived luminosity distance-redshift relation is perfectly consistent with the measurements of type Ia supernovae. Therefore, with the new luminosity distance-redshift relation, the SNeIa measurements do not show the existence of dark energy.

The analysis and measurements for the structure and weak lensing of the CMB might not be accurate enough as were thought to provide an independent check or evidence on the

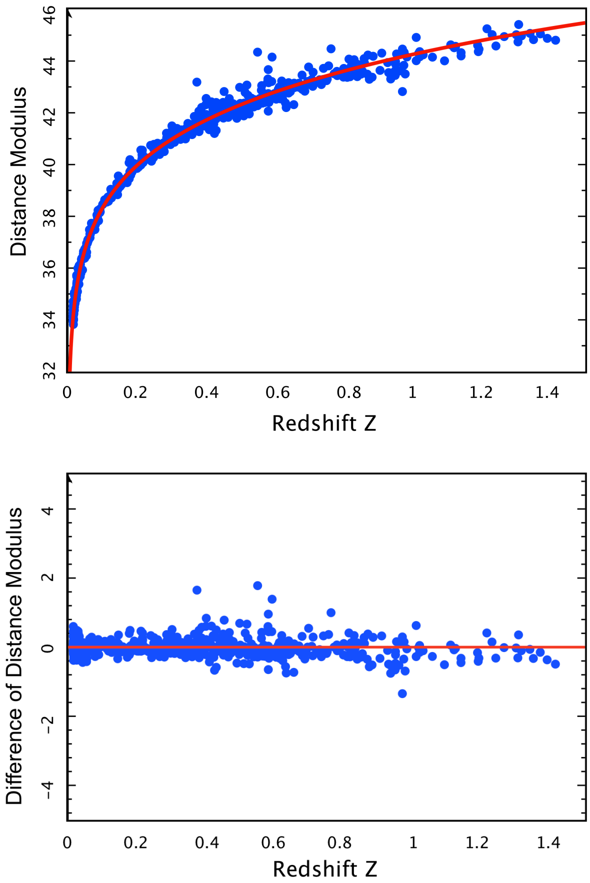


Fig. 3: Luminosity distance-redshift relation of type Ia supernovae. Blue dots are measurements credited by the Union2 compilation of 580 SNeIa data from Supernova Cosmology Project. Red lines are analytical results from this study. The upper panel plots the distance modulus as a function of redshift, while the lower panel plots the distance modulus difference between the measurement data and theoretical results.

existence of dark energy [29-30]. Recently, Sawangwit and Shank [31-32] looked at the CMB observations and find the errors in the data to be much larger than previously thought. The CMB power spectrum is very sensitive to the beam profiles. If their results are further confirmed to be correct, then it will also become less likely that dark energy dominates the universe.

## 4 Summary

The luminosity distance-redshift relation that we previously applied to connect the models with the SNeIa measurements is an approximate expression only valid for nearby objects. This is because that the traditional derivation of the $D_L - Z$ relation has the following two defects: (1) the light emitting angle is about equal to the light incident angle, which is not true for light from a distant source object according to the view of the observer on the earth, and (2) the redshift of light and the increase of time interval doubly reduce the energy flux of the received light, which is physically incorrect because the redshift of light is caused by the increase of time interval. The electromagnetic wave theory of light also supports that the apparent luminosity is reduced only by one redshift factor due to the time interval increase. We have corrected these defects and derived a new relationship between luminosity distance and redshift with a factor of $\sqrt{1+Z}$. With this new $D_L - Z$ relation, we have perfectly explained the SNeIa measurements according to the standard cosmological model without dark energy ($\Lambda = 0$). Therefore, we can conclude that the universe has not accelerated and does not need the dark energy at all. The luminosity distance-redshift relation often used previously is only valid for nearby objects and thus the luminosity distances of all distant type Ia supernovae had been underestimated. This study provides us a possible solution to the mystery of dark energy.


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