Mass-Radius Relations of Z and Higgs-Like Bosons

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Relations between the rest mass and the effective radius are deduced for the Z boson and the experimentally discovered Higgs-like boson, in terms of a revised quantum electrodynamic (RQED) theory. The latter forms an alternative to the Standard Model of elementary particles. This results in an effective radius of the order of 10^{-18} m for the Z boson, in agreement with accepted data. A composite model for the Higgs-like boson is further deduced from the superposition of solutions represented by two Z bosons. This model satisfies the basic properties of the observed Higgs-like particle, such as a vanishing charge and spin, a purely electrostatic and strongly unstable state, and an effective radius of about 10^{-18} m for a rest mass of 125 GeV.

1 Introduction

Recently an elementary particle has been discovered at the projects ATLAS [1] and CMS [2] of CERN, being unstable, having vanishing net electric charge and spin, and a rest mass of 125 GeV. This discovery was made in connection with a search for the Higgs boson and its theoretical base given by the Standard Model of an empty vacuum state.

Being distinguished from the latter model, a revised quantum electrodynamic (RQED) theory has been elaborated [3], as founded on the principle of a non-empty vacuum state. It is supported by the quantum mechanical Zero Point Energy [4] and the experimentally verified Casimir force [5]. This relativistic and gauge invariant theory of broken symmetry is based on a nonzero electric field divergence in the vacuum, in combination with a vanishing magnetic field divergence due to the non-existence of observed magnetic monopoles.

Among the subjects being treated by RQED theory, this report is devoted to the mass-radius relation obtained for the Z boson, and to that associated with a model of the Higgs-like boson. This provides an extension of an earlier analysis on a Higgs-like particle [6].

2 Particle with vanishing net electric charge

Due to the RQED theory of axisymmetric particle-shaped steady states with rest mass, a separable generating function

$$F(r,\theta) = CA - \phi = G_0 G(\rho,\theta), \quad G = R(\rho)\cdot T(\theta) \tag{1}$$

can be introduced in a spherical frame (r,θ,φ) of reference [3]. There is an electrostatic potential ϕ and an electric charge density $\bar{\rho} = \varepsilon_0 \operatorname{div} \mathbf{E}$, a current density $\mathbf{j} = (0,0,C\bar{\rho})$ with $\mathbf{C}^2 = c^2$ and $C = \pm c$ representing the two spin directions along φ, and a magnetic vector potential $\mathbf{A} = (0,0,A)$. A dimensionless radial coordinate $\rho = r/r_0$ is introduced with a characteristic radius r_0, and a dimensionless generating function G with the characteristic amplitude G_0.

As based on the function (1), the general forms of the potentials and the charge density become

$$CA = -(\sin\theta)^2 DF\,, \tag{2}$$

$$\phi = -\left[1 + (\sin\theta)^2 D\right] F\,, \tag{3}$$

$$\bar{\rho} = -\frac{\varepsilon_0}{r_0^2\rho^2} D\left[1 + (\sin\theta)^2 D\right] F\,, \tag{4}$$

where the operators are

$$D = D_\rho + D_\theta$$
$$D_\rho = -\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial}{\partial\rho}\right) \qquad D_\theta = -\frac{\partial^2}{\partial\theta^2} - \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}\,. \tag{5}$$

Since the analysis will be applied to the special class of particles with vanishing net electric charge, such as the Z and Higgs-like bosons, the radial part R of the function (1) has to be convergent at the origin $\rho = 0$, and a polar part T is chosen having top-bottom symmetry with respect to the equatorial plane $\theta = \pi/2$. This is due to earlier performed basic deductions [3].

Due to the non-zero electric field divergence, there are local intrinsic charges even when the net integrated charge vanishes. For a convergent generating function the total integrated energy W can either be expressed in terms of the field energy density

$$w_f = \frac{1}{2}\varepsilon_0\left(\mathbf{E}^2 + c^2\mathbf{B}^2\right) \tag{6}$$

or of the source energy density

$$w_s = \frac{1}{2}\bar{\rho}\,(\phi + CA) \tag{7}$$

from which

$$W = \int w_f\, dV = \int w_s\, dV. \tag{8}$$

We shall use the option (7) for which the local contribution to the particle mass becomes

$$dm_0 = \frac{w_s}{c^2}\, dV \tag{9}$$

and that related to the angular momentum (spin) becomes

$$ds_0 = Cr\,(\sin\theta)\; dm_0 \tag{10}$$

for a volume element $dV = 2\pi r^2(\sin\theta)\, d\theta dr$ in a spherical frame.

A generating function being convergent both at $\rho = 0$ and at large ρ, and having top-bottom symmetry, is finally chosen through the form

$$R = \rho^{\gamma} \cdot e^{-\rho}, \qquad T = (\sin\theta)^{\alpha}, \tag{11}$$

where $\gamma \geqslant 1$ and $\alpha \geqslant 1$. The part R then increases to a maximum at the effective radius $\hat{r} = \gamma r_0$ after which it drops steeply towards zero at large ρ.

3 Model of a Z boson

A Z boson is first considered, having zero net electric charge, spin $h/2\pi$, a rest mass of 91 GeV, and an effective radius of about 10^{-18} m according to given data [7].

From (1)–(5), (8), (9) and (11) the product of the mass m_{0Z} and the effective radius $\hat{r}_Z = \gamma r_{0Z}$ becomes

$$\hat{r}_Z m_{0Z} = \pi \left(\varepsilon_0/c^2\right) r_{0Z}^2 G_0^2 \gamma J_{mZ}, \tag{12}$$

where

$$J_{mZ} = \int_0^{\infty}\int_0^{\pi} f\, g_Z\, d\rho\, d\theta \tag{13}$$

and

$$f = -(\sin\theta)\, D\left[1 + (\sin\theta)^2 D\right] G, \tag{14}$$

$$g_Z = -\left[1 + 2\,(\sin\theta)^2 D\right] G. \tag{15}$$

The spin is further given by

$$s_{0Z} = \pi\varepsilon_0 \left(C/c^2\right) r_{0Z}^2 G_0^2 J_{sZ} = \pm h/2\pi \tag{16}$$

where

$$J_{sZ} = \int_0^{\infty}\int_0^{\pi} \rho\,(\sin\theta)\, f\, g_Z\, d\rho\, d\theta. \tag{17}$$

Combination of (12)–(17) yields

$$\hat{r}_Z m_{0Z} = \frac{h}{2\pi c}\frac{\gamma J_{mZ}}{J_{sZ}}. \tag{18}$$

This relates the mass to the effective radius, in a way being dependent on the profile shape of the generating function:

- A numerical analysis of the $1 \leqslant \gamma \leqslant 10$ and $1 \leqslant \alpha \leqslant 10$ cases, results in the large ranges $17.7 \leqslant J_{mZ} \leqslant 9.01 \times 10^{15}$ and $39.8 \leqslant J_{sZ} \leqslant 1.83 \times 10^{16}$ of the amplitudes J_{mZ} and J_{sZ}. The last factor of the right-hand member in (18) stays however within the limited range of $0.445 \leqslant (\gamma J_{mZ}/J_{sZ}) \leqslant 0.904$.
- In the asymptotic cases $\gamma \gg \alpha \gg 1$ and $\alpha \gg \gamma \gg 1$ the values of $(\gamma J_{mZ}/J_{sZ})$ become 15/38 and 1, respectively. This is verified in an earlier analysis [8].
- In spite of the large variations of J_{mZ} and J_{sZ} with the profile shape, the factor $\gamma J_{mZ}/J_{sZ}$ thus has a limited variation within a range of about 0.4 to 1.

For the present deduced model, the rest mass of 91 GeV then results in an effective radius in the range of 0.87×10^{-18} to 2.2×10^{-18} m. This is consistent with the given value of $\hat{r}_Z$.

For the expressions (2) and (3) combined with the form (11) can finally be seen that there is a moderately large deviation from a state $\mathbf{E}^2 = c^2\mathbf{B}^2$ of equipartition between the electrostatic and magnetostatic particle energies.

4 Model of a Higgs-like boson

One of the important reactions being considered in the experiments at CERN is the decay of the observed Higgs-like boson into two Z bosons, and further into four leptons. Since the Higgs-like boson was found to have a mass of 125 GeV, and the Z bosons have masses of 91 GeV each, an extra contribution of 57 GeV is required for the decay into the Z bosons. It can then be conceived that this extra energy is "borrowed" from the Heisenberg uncertainty relation when the entire decay process takes place in a very short time. At least one of the involved Z bosons then behaves as a virtual particle. In this connection is also observed that the magnitude of the Higgs-like boson mass has not been predicted through the theory by Higgs [9].

With the decay process in mind, a relation will now be elaborated between the mass and the effective radius of the Higgs-like boson. Then it has first to be observed that a relation similar to equation (18) cannot be straightforwardly deduced. This is because the Higgs-like boson has no spin, and its related effective radius can on account of the required extra energy not become identical with that of a single Z boson.

Solutions for models of massive individual bosons and leptons are available from RQED theory. The field equations are linear, and these solutions can be superimposed to form a model of a Higgs-like particle having vanishing charge and spin. It can be done in terms of four leptons or two Z bosons. Choosing the latter option [6], superposition of the potentials (2) and (3) for two modes with opposite spin directions results in a composite Higgs-like mode with zero charge and spin but nonzero rest mass. This mode has no magnetic field, is purely electrostatic, and is thus expected to be highly unstable. In analogy with the deductions (1)–(11), the corresponding integrated mass m_{0H} becomes

$$\hat{r}_H m_{0H} = \pi \left(\varepsilon_0/c^2\right) r_{0H}^2 G_0^2 \gamma J_{mH} \tag{19}$$

with the effective radius $\hat{r}_H \neq \hat{r}_Z$ and

$$J_{mH} = \int_0^{\infty}\int_0^{\pi} f\, g_H\, d\rho\, d\theta. \tag{20}$$

Here f is still obtained from (14) and

$$g_H = -2\left[1 + (\sin\theta)^2 D\right] G = g_Z - G \tag{21}$$

with g_Z given by (15). Combination of (19) and (12) yields

$$\frac{\hat{r}_H}{\hat{r}_Z} = \frac{r_{0H}}{r_{0Z}} = \frac{m_{0H}}{m_{0Z}} \frac{J_{mZ}}{J_{mH}} . \qquad (22)$$

The dependence on the profile shape of the generating function is as follows:

- Numerical analysis in the ranges $1 \leqslant \gamma \leqslant 10$ and $1 \leqslant \alpha \leqslant 10$ results in the amplitude variations $138 \leqslant J_{mZ} \leqslant 9.8 \times 10^{15}$ and $287 \leqslant J_{mH} \leqslant 1.8 \times 10^{16}$, but their ratio is strongly limited to $2.03 \leqslant J_{mH}/J_{mZ} \leqslant 2.20$.
- From expression (21) at large γ and α can further be seen that J_{mH}/J_{mZ} approaches the asymptotic value 2.

With these evaluations, and the experimentally determined masses $m_{0Z} = 91$ GeV and $m_{0H} = 125$ GeV, the effective radius $\hat{r}_H$ of the Higgs-like boson comes from (22) out to be in the range 0.54×10^{-18} to 1.5×10^{-18} m.

5 Summary

The present model of the Z boson leads to an effective radius of the order of 10^{-18} m, in agreement with given data. This can be taken as support of the present theory.

Concerning the present model of a Higgs-like boson, the following results should be observed:

- An imagined "reversal" of the decay of a Higgs-like boson into two Z bosons initiates the idea of superimposing two Z boson modes to form a model of such a particle. The resulting composite particle solution is consistent with the point made by Quigg [7] that the Higgs is perhaps not a truly fundamental particle but is built out of as yet unobserved constituents.
- The present model of a Higgs-like boson satisfies the basic properties of the particle observed at CERN. It has a vanishing electric charge and spin, a nonzero rest mass, and is unstable due to its purely electrostatic nature.
- The present theory finally results in an effective radius of the order of 10^{-18} m for the experimentally detected Higgs-like particle having a rest mass of 125 GeV, and vice versa.

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