Nuclear Potential Energy Surfaces and Critical Point Symmetries within the Geometric Collective Model

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The critical points of potential energy surface (PES's) of the limits of nuclear structure harmonic oscillator, axially symmetric rotor and deformed γ-soft and discussed in framework of the general geometric collective model (GCM). Also the shape phase transitions linking the three dynamical symmetries are studied taking into account only three parameters in the PES's. The model is tested for the case of $^{238}_{92}$U , which shows a more prolate behavior. The optimized model parameters have been adjusted by fitting procedure using a simulated search program in order to reproduce the experimental excitation energies in the ground state band up to 6^+ and the two neutron separation energies.

1 Introduction

Shape phase transitions from one nuclear shape to another were first discussed in framework of the interacting boson model (IBM) [1]. The algebraic structure of this model is based upon U(6) and three dynamical symmetries arise involving the sub algebras U(5), SU(3) and O(6). There have been numerous recent studies of shape phase transitions between the three dynamical symmetries in IBM [2–9]. The three different phases are separated by lines of first-order phase transition, with a singular point in the transition from spherical to deformed γ-unstable shapes, which is second order. In the usual IBM-1 no triaxial shape appears.

Over the years, studies of collective properties in the framework of geometric collective model (GCM) [3, 10–12] have focused on lanthanide and actinide nuclei. In GCM the collective variables β (the ellipsoidal deformation) and γ (a measure of axial asymmetry) are used. The characteristic nuclear shapes occuring in the GCM are shown in three shapes which are spherical, axially symmetric prolate deformed (rotational) and axial asymmetry (γ -unstable). The shape phase transitions between the three shapes have been considered by the introduction of the critical point symmetries E(5) [13] and X(5) [14]. The dynamical symmetry E(5) describe the phase transition between a spherical vibrator (U(5)) and γ-soft rotor (O(6)) and the X(5) for the critical point of the spherical to axially deformed (SU(3)) transition. Also the critical point in the phase transition from axially deformed to triaxial nuclei, called Y(5) has been analyzed [15].

The main objective of this study is to analyze the importance of the critical points in nuclear shapes changes. The paper is organized as follows. In sec. 2 we survey the framework of the GCM and the method to analyze the PES's in terms of the deformation variables β and γ. In section 3 we study the behavior of the critical point. In section 4 we present the numerical result for realistic case to even-even ^{238}U nucleus and give some discussions. Finally in section 5, the conclusions of this work are made.

2 Potential Energy Surfaces in Geometric Collective Model

We start by writing the GCM Hamiltonian in terms of irreducible tensor operators of collective coordinates α's and conjugate momenta π as:

$$H = \frac{1}{2B_2}[\pi\times\pi]^{(0)} + C_2[\alpha\times\alpha]^{(2)} + C_3[[\alpha\times\alpha]^{(2)}\times\alpha]^{(0)} + C_4[\alpha\times\alpha]^{(0)}[\alpha\times\alpha]^{(0)} \quad (1)$$

where B_2 is the common mass parameter of the kinetic energy term and C_2, C_3 and C_4 are the three stiffness parameters of collective potential energy. They are treated as adjustable parameters which have to be determined from the best fit to the experimental data, level energies, B(E2) transition strengths and two-neutron separation energy. The corresponding collective potential energy surface (PES) is obtained by transforming the collective coordinate $a_{2\nu}$ into the intrinsic coordinate $a_{2\nu}$. To separate the three rotational degree of freedom one only has to set

$$\alpha_{2\mu} = \sum_{\nu} D^{*2}_{\mu\nu}(\omega)a_{2\nu}. \quad (2)$$

Since the body axes are principle axes, the products of inertia are zero, which implies that $a_{21} = a_{2-1} = 0$ and $a_{22} = a_{2-2}$. The two remaining variables a_{20} and a_{22}, to gather with Eulerian angles ω, would completely describe the system replacing the five $\alpha_{2\mu}$. However, there is rather more direct physical significance in the variables β and γ defined by

$$a_{20} = \beta\cos\gamma \quad (3)$$

$$a_{22} = \frac{1}{\sqrt{2}}\beta\sin\gamma \qquad (4)$$

where β is a measure of the total deformation of the nucleus and γ indicate the deviations from axial symmetry. In terms of such intrinsic parameters, we have

$$[\alpha\times\alpha]^{(0)} = \frac{\beta^2}{\sqrt{5}} \qquad (5)$$

$$[[\alpha\times\alpha]^{(2)}\times\alpha]^{(0)} = -\sqrt{\frac{2}{35}}\beta^3\cos 3\gamma. \qquad (6)$$

The PES belonging to the Hamiltonian (1) then reads

$$E(\beta,\gamma) = C_2\frac{1}{\sqrt{5}}\beta^2 - C_3\sqrt{\frac{2}{35}}\beta^3\cos 3\gamma + C_4\frac{1}{5}\beta^4. \qquad (7)$$

The values of β and γ are restricted to the intervals $0\le\beta\le\infty$, $0\le\gamma\le\pi/3$. In other words the $\pi/3$ sector of the $\beta\gamma$ plane is sufficient for the study of the collective PES's.

3 Critical Point Symmetries

Minimization of the PES with respect to β gives the equilibrium value β_m defining the phase of the system. $\beta_m = 0$ corresponding to the symmetric phase and $\beta_m \neq 0$ to the broken symmetry phase. Since γ enters the potential (7) only through the $\cos 3\gamma$ dependence in the cubic term, the minimization in this variable can be performed separately. The global minimum is either at $\gamma_m = 0(2\pi/3, 4\pi/3)$ for $C_3 > 0$ or at $\gamma_m = \pi/3(\pi, 5\pi/3)$ for $C_3 < 0$. The second possibility can be expected via changing the sign of the corresponding β_m and simultaneously setting $\gamma_m = 0$. The phase can be described as follows:

1. For $C_3^2 < \frac{14C_2|C_4|}{\sqrt{5}}$, phase with $\beta_m = 0$ interpreted as spherical shape.
2. For $C_3^2 < \frac{14C_2|C_4|}{\sqrt{5}}, C_3 > 0$, phase with $\beta_m > 0, \gamma_m = 0$ interpreted as prolate deformed shape.
3. For $C_3^2 < \frac{14C_2|C_4|}{\sqrt{5}}, C_3 < 0$, phase with $\beta_m > 0, \gamma_m = \pi/3$ interpreted as oblate deformed shape.

For β non-zero the first derivative of equation (7) must be zero and the second derivative positive. This gives

$$\begin{aligned}\frac{4}{5}C_4\beta^2 - 3\sqrt{\frac{2}{35}}C_3\beta^3\cos 3\gamma + \frac{2}{\sqrt{5}}C_2 &= 0\\ \frac{12}{5}C_4\beta^2 - 6\sqrt{\frac{2}{35}}C_3\beta^3\cos 3\gamma + \frac{2}{\sqrt{5}}C_2 &> 0 \qquad (8)\end{aligned}$$

The solution of equation (8), yields $\beta_{\pm} = \frac{3}{4}\sqrt{\frac{5}{14}}(1\pm r)e$ with $r = \sqrt{1-d}$, $d = \frac{112}{9\sqrt{5}}\frac{C_2C_4}{C_3^2}$ and $e = \frac{C_3}{C_4}$.

The minimum values of the potential are

$$E(\beta) = -\frac{135}{50176}(r\pm 1)^3(3r\mp 1)f \qquad (9)$$

with $f = \frac{C_3^4}{C_4^3}$.

For $d > 1$ there is only one minimum located at $\beta = 0$. For $0 < d < 1$, minima are present both at non-zero β and at $\beta = 0$, with the deformed minimum lower $0 < d < 8/9$ and the undeformed minimum lower for $8/9 < d < 1$. For $d < 0$, the potential has both a global minimum and a saddle point at non-zero β. For harmonic vibrator shape $C_3 = C_4 = 0$, this yields

$$E(\beta) = \frac{C_2}{\sqrt{5}}\beta^2, \qquad C_2 > 0. \qquad (10)$$

For γ-unstable shape, the solution for$\beta \neq 0$ are obtained if we set $C_3 = 0$ in equation (8). Then equation (8) gives

$$\frac{4}{5}C_4\beta^2 + \frac{2}{\sqrt{5}}C_2 = 0$$

or

$$\beta = \pm\sqrt{\frac{-\sqrt{5}}{2}\frac{C_2}{C_4}} \simeq \pm 1.057\sqrt{\frac{-C_2}{C_4}};$$

this requires C_4 and C_2 to have opposite sign. Since C_4 must be positive for bound solutions C_2 must be negative in deformed γ-unstable shape. That is the spherical — deformed phase transition is generated by a change in sign of C_2, while the prolate-oblate phase is corresponding to changing the sign of C_3. For symmetric rotor one needs with both a deformed minimum in β and a minimum in γ, at $\gamma = 0$ for prolate or $\gamma = \pi/3$ for oblate. For prolate shape this requires $C_3 > 0$, such a potential has a minimum in β at $\beta_{\pm}$ equation (7). For $\gamma = 0$ (to study the β-dependence), and providing that $C_2 > 0$ and $C_3 > 0$, then the critical point is located at $C_3^2 < 14C_2|C_4|/\sqrt{5}$.

In Fig. (1a) a typical vibrator is given, the minimum of the PES is at $\beta = 0$ and therefore the ground state is spherical. In

Table 1: The GCM parameters for shape-phase transition (a) from vibrator to rotor (b) from rotor to γ-soft.

	C_2	C_3	C_4
set (a)	1	0	0
	-0.25	0.7	10
	-1	1	20
	-2.5	1.7	29
set (b)	-3	2	40
	-4.2	1.5	80
	-4.5	1	120
	-5	0	170

Fig. (1b) a typical axially deformed prolate is given, where the minimum it as $\beta \neq 0$ and the ground state is deformed. In Fig. (1c) a case of γ-unstable shape is illustrated. Fig. (2a) gives the PES's calculated with GCM as a function of the shape poor rotor β for shape phase transition from spherical to prolate deformed and in Fig. (2b) from rotor to γ-soft. The model parameters are listed in Table (1).

For simplicity we write equation (7) when $\gamma = 0$ in form

$$E(\beta) = A_2\beta^2 + A_3\beta^3 + A_4\beta^4. \tag{11}$$

The extremism structure of the PES depends only upon the value A_2 as summarized in Table (2) and Fig. (3). For $A_2 < 0$ the potential has both a global minimum and a saddle point at non-zero β. For $A_2 > 0$, minima are present at both $\beta \neq 0$ and $\beta = 0$ with the deformed minimum lower for $A_2 = 109.066$ and the undeformed minimum lower for $A_2 = 161.265$. For $A_2 = 22.6$ there is only one minimum located at $\beta = 0$.

4 Application to $^{238}_{92}$U

We applied the GCM to the doubly even actinide nucleus ^{238}U. The optimized model parameter was adjusted by fit-

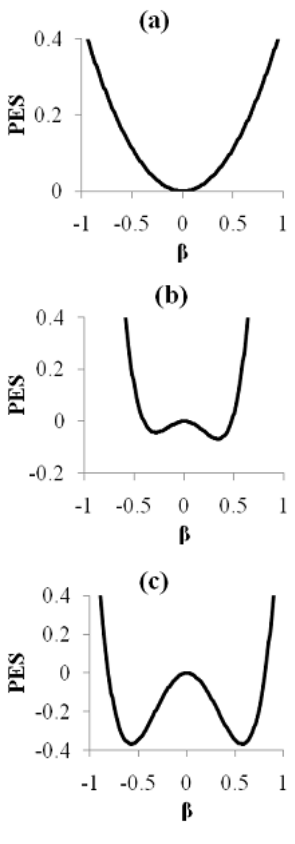

Fig. 1: Potential energy surface (PES's) in framework of GCM for three different shapes (a) harmonic vibrator shape ($C_2 = 1$, $C_3 = 0$, $C_4 = 0$) (b) strongly axially deformed prolate shape ($C_2 = -2.5$, $C_3 = 1.7$, $C_4 = 29$) (c) γ-unstable shape ($C_2 = -5$, $C_3 = 0$, $C_4 = 17$).

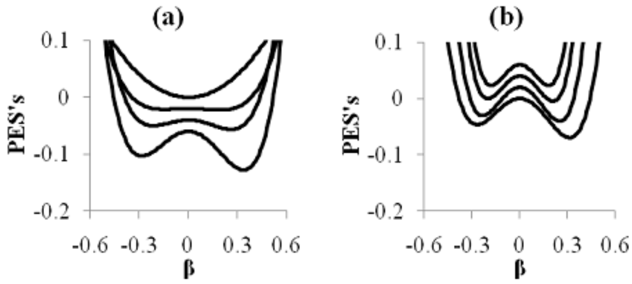

Fig. 2: Potential energy surface (PES's) in framework of GCM for two different shape transitions (a) from vibrator to rotor (b) from rotor to γ-soft rotor the set of parameters are listed in Table (1).

Table 2: Set of control parameters of the GCM to describe the nature of the critical points.

A_2	A_3	A_4
22.600	-1.120	0.234
66.412	-294.869	368.217
161.265	-935.148	1148.890
85.714	-573.709	960.000
109.066	-881.661	1603.589
0.000	-152.991	387.884
-15.581	-48.791	214.854
-22.098	-3.286	137.500

ting procedure using a computer simulated search program in order to reproduce some selected experimental excitation energies ($2^+_1, 4^+_1, 6^+_1$) and the two neutron separation energies. The PES versus the deformation parameter β for ^{238}U is illustrated in Fig. (4). The figure show that ^{238}U exhibit a deformed prolate shape.

5 Conclusion

In this study we used the GCM to produce the PES's to investigate the occurrence of shape phase transitions. The critical point symmetries are obtained. The validity of the model is examined for ^{238}U. A fitting procedure was proposed to deforming the parameters of the geometric collective Hamiltonian for the axially symmetric deformed rotor.

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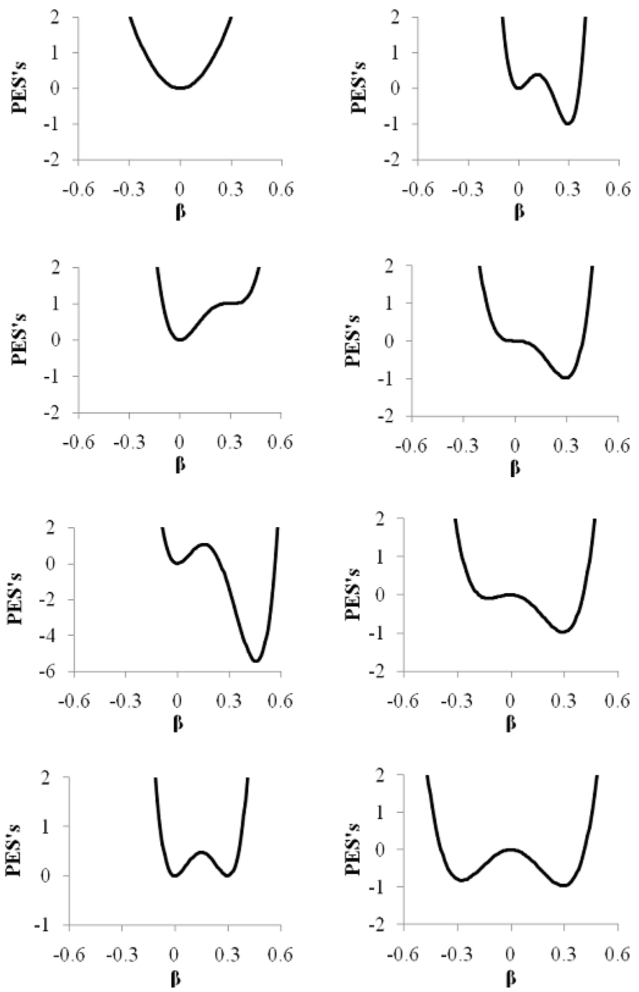

Fig. 3: Different shapes of PES's by varying the control parameters listed in Table (2).

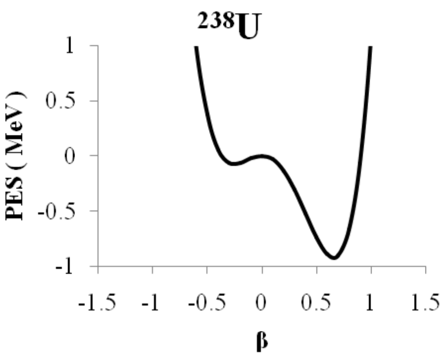

Fig. 4: The Potential energy surface (PES) as a function of deformation parameter β for ^{238}U and a cut through $\gamma = 0$ and $\gamma = \Pi/3$ are given with the parameters ($C_2 = -6.23928$, $C_3 = 18.63565$, $C_4 = 41.51437$).

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