LETTERS TO PROGRESS IN PHYSICS

Some Insights on the Nature of the Vacuum Background Field in General Relativity

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In our publication "Vacuum Background field in General Relativity" (*Progress in Physics*, 2016, v. 12, issue 4, 313–317) we introduced a kind of "relic" field permanently filling the empty space-time. This proved to be a necessary ingredient to formulate a true vector describing the gravitational field arising from matter, in contrast to the awkward pseudo-tensor usually suggested to ensure the conservation of the energy-momentum in the field equations. In this short paper, we give this field a mathematical description in terms of geodesics.

The background field that persists in vacuum devoid of any matter or energy, finds a physical meaning if we consider the *Landau-Raychaudhuri equation* for a congruence of non-intersecting timelike unit vectorial field $\boldsymbol{X}$, ($X_a X^a = 1$), i.e.:

$$R^a_{\ b} X_a X^b = -{}^\circ X^a_{\ ;a} - \omega_{ab}\,\omega^{ab} + \sigma_{ab}\,\sigma^{ab} + {}^\circ\theta + \frac{1}{3}\,\theta^2, \qquad (1)$$

where $^\circ$ means differentiation with respect to proper time τ. In the scalar ζ which is the *Lagrangian density of the vacuum background field*

$$\zeta = \sqrt{-g}\,\nabla_a\,\kappa^a \qquad (2)$$

we set up

$$\nabla_a\,\kappa^a = \theta^2, \qquad (3)$$

where θ is the *space time volume expansion* characterizing this background field through

$$\theta = X^a_{\ ;a} = h^{ab}\theta_{ab} \qquad (4)$$

with the expansion tensor $\theta_{ab} = h^c_a h^d_b X_{(c;d)}$ ($h_{ac} = g_{ac} - X_{ac}$ is the projection tensor).

The formula of $X_{(c;d)}$ can be regarded as measuring the rate of change of the space-time 4-volume, i.e. either expansion if positive, or contraction if negative.

The form (3) has been chosen so as to preserve the integrity of the gravity tensor equation irrespective of the sign of θ.

In our case (absence of energy/matter), the background field obviously follows a contraction process (negative expansion) of space-time, and the Landau-Raychaudhuri reduces to

$$^\circ\theta = R^a_{\ b} X_a X^b - \sigma_{ab}\,\sigma^{ab} - \frac{1}{3}\,\theta^2 < 0 \qquad (5)$$

(since the vorticity tensor ω^{ab} induces expansion, while the shear tensor σ^{ab} induces contraction and the geodesic equation $^\circ X^a_{\ ;a}$ is zero).

$R^a_{\ b} X_a X^b$ (sometimes referred to as the Raychaudhuri scalar) is always positive ensuring that the Strong Energy Condition (SEC) is not violated when energy/matter is there. Therefore, we are left with the inequality

$$^\circ\theta \leqslant \frac{1}{3}\,\theta^2. \qquad (6)$$

Integrating it with respect to the proper time τ yields

$$\theta^{-1} \geqslant \theta_1^{-1} + \frac{1}{3}\,\tau\,, \qquad (7)$$

where θ_1 is the initial value which can be positive to start with, but very soon after the short expansion, it is followed by re-collapse. The mathematical fate of (timelike) geodesics is a final focusing to a caustic ($\theta \to -\infty$) after a finite proper time of at most

$$\tau \leqslant \frac{3}{\theta_1} \qquad (8)$$

after the measurement of the initial value.

Such a state is called *geodesic incompleteness* which is a notion introduced by Hawking-Penrose, to describe (or not !) a geodesics path of observers through space-time that can only be extended for a finite time as measured by an observer travelling along one.

Presumably, at the end of the geodesic, the observer has fallen into a "kink" or encountered some other pathology at which the laws of General Relativity breakdown.

As Landau pointed out, in a *synchronous comoving frame of reference* attached to a homogeneous fluid, such a singularity can be removed by the introduction of a pressure which tends to substantiate our space-time contraction hypothesis.

All these contribute to our impossibility to give a full description of the vacuum background field. In a sense, this marks the lowest horizon level of the space-time.

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