On Time Dilation, Space Contraction, and the Question of Relativistic Mass

Pierre A. Millette

E-mail: PierreAMillette@alumni.uottawa.ca, Ottawa, Canada

In this paper, we revisit the question of relativistic mass to clarify the meaning of this concept within special relativity, and consider time dilation and length contraction in more detail. We see that "length contraction" is a misnomer and that it should really be named "space contraction" to avoid confusion, and demonstrate the complementary nature of time dilation and space contraction. We see that relativistic mass is dependent on the difference in velocity v between an object's proper frame of reference that is at rest with the object and the frame of reference from which it is observed. We show that the inertial mass of a body is its proper mass while the relativistic mass m^* is in effect an effective mass. We find that relativistic mass results from dealing with dynamic equations in local time t in a frame of reference moving with respect to the object of interest, instead of the invariant proper time τ in the frame of reference at rest with the object. The results obtained are in agreement with the Elastodynamics of the Spacetime Continuum.

1 Introduction

The concept of relativistic mass has been a part of special relativistic physics since it was first introduced by Einstein [1, 2] and explored by the early relativists (see for example [3, 4]). Other terminology is also used for relativistic mass, representing the users' perspective on the concept. For example, Aharoni [5] refers to it as the "relative mass", while Dixon [6] refers to it as "apparent mass". Oas [7] and Okun [10] provide good overviews on the development of the historical use of the concept of relativistic mass. Oas [8] has prepared a bibliography of published works where the concept is used and where it is ignored.

There is no consensus in the physics community on the validity and use of the concept of relativistic mass. Some consider relativistic mass to represent an actual increase in the inertial mass of a body [12]. However, there have been objections raised against this interpretation (see Taylor and Wheeler [14], Okun [9–11], Oas [7]). The situation seems to arise from confusion on the meaning of the special relativistic dynamics equations. In this paper, we revisit the question of relativistic mass to clarify the meaning of this concept within special relativity, in light of the Elastodynamics of the Spacetime Continuum (*STCED*) [18, 19].

2 Relativistic mass depends on the frame of reference

The relativistic mass m^* is given by

$$m^* = \gamma m_0\,, \tag{1}$$

where

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}}\,, \tag{2}$$

$\beta = v/c$ and m_0 is the rest-mass or proper mass which is an invariant. Some authors [11] suggest that rest-mass should be denoted as m as this is the real measure of inertial mass. The relativistic mass of an object corresponds to the total energy of an object (invariant proper mass plus kinetic energy). The first point to note is that the relativistic mass is the same as the proper mass in the frame of reference at rest with the object, *i.e.* $m^* = m_0$ for $v = 0$. In any other frame of reference in motion with velocity v with respect to the object, the relativistic mass will depend on v according to (1).

For example, when the relativistic mass of a cosmic ray particle is measured[†] in an earth lab, it depends on the speed of the particle measured with respect to the earth lab. Similarly for a particle in a particle accelerator, where its speed is measured with respect to the earth lab. The relativistic mass of the cosmic ray particle measured from say a space station in orbit around the earth or a spaceship in transit in space would depend on the speed of the particle measured with respect to the space station or the spaceship respectively.

We thus see that relativistic mass is an effect similar to length contraction and time dilation in that it is dependent on the difference in velocity v between the object's frame of reference and the frame of reference from which it is measured. Observers in different moving frames will measure different relativistic masses of an object as there is no absolute frame of reference with respect to which an object's speed can be measured.

3 Time dilation and space contraction

To further understand this conclusion, we need to look into time dilation and length contraction in more detail. These special relativistic concepts are often misunderstood by physicists. Many consider these changes to be actual physical changes, taking the Lorentz-Fitzgerald contraction and the time dilation effect to be real.

[†] what is measured is the energy of the particle, not its mass.

For example, John Bell in [15] relates the problem of the thread tied between two spaceships and whether the thread will break at relativistic speeds due to length contraction. He insists that it will – he relates how "[a] distinguished experimental physicist refused to accept that the thread would break, and regarded my assertion, that indeed it would, as a personal misinterpretation of special relativity". Bell appealed to the CERN Theory Division for arbitration, and was dismayed that a clear consensus agreed that the thread would not break, as indeed is correct. As the number of special relativistic "paradoxes" attest, many physicists, scientists and engineers have similar misunderstandings, not clearly understanding the concepts.

This situation arises due to not realizing that v is the difference in velocity between an object's frame of reference and the frame of reference from which it is measured, not an absolute velocity, as discussed in the previous section 2. In a nutshell, time dilation and length contraction are apparent effects. In the frame of reference at rest with an object that is moving at relativistic speeds with respect to another frame of reference, there is no length contraction or time dilation.

The proper time in the frame of reference at rest with the object is the physical time, and the length of the object in the frame of reference at rest with the object is the physical length – there is no time dilation or length contraction. These are observed in other frames of reference moving with respect to that object and are only apparent dilations or contractions perceived in those frames only. Indeed, observers in frames of reference moving at different speeds with respect to the object of interest will see *different* time dilations and length contractions. These cannot all be correct – hence time dilation and length contraction are apparent, not real.

This can be demonstrated to be the case from physical considerations, and in so doing, we clarify further the nature of length contraction. Petkov [13] provides graphically a physical explanation of time dilation and length contraction, based on Minkowski's 1908 paper [16] where the latter first introduced the concept of a four-dimensional spacetime and the description of particles in that spacetime as worldlines. Worldlines of particles at rest are vertical straight lines in a *space−ct* diagram, while particles moving at a constant velocity v are oblique lines and accelerated particles are curved lines.

The basic physical reason for these effects can be seen from the special relativistic line element (using x to represent the direction of propagation and $c = 1$)

$$d\tau^2 = dt^2 - dx^2\,. \tag{3}$$

One sees that for a particle at rest, the vertical straight line in a *space−ct* diagram is equivalent to

$$d\tau^2 = dt^2\,, \tag{4}$$

which is the only case where the time t is equivalent to the proper time τ (in the object's frame of reference). In all other cases, in particular for the oblique line in the case of constant velocity v, (3) applies and there is a mixing of space x and time t, resulting in the perceived special relativistic effects observed in a frame of reference moving at speed v with respect to the object of interest.

Loedel diagrams [17], a variation on *space−ct* diagrams allowing to display the Lorentz transformation graphically, are used to demonstrate graphically length contraction, time dilation and other special relativistic effects in problems that involve two frames of reference. Figs. 1 and 2, adapted from Petkov's Figs. 4.18 [12, p. 86], and 4.20 [12, p. 91] respectively, and Sartori's Fig. 5.15 [17, p. 160], provide a graphical view of the physical explanation of time dilation and length contraction respectively.

From Fig. 1, we see that $\Delta t' > \Delta t$ as expected – the moving observer sees time interval $\Delta t'$ of the observed object to be dilated, while the observed object's time interval Δt is actually the physical proper time interval $\Delta\tau$. From Fig. 2, we see that space distance measurements, *i.e.* space intervals, $\Delta x' < \Delta x$ as expected – the moving observer sees space interval $\Delta x'$ of the observed object to be contracted, while the observed object's space interval Δx is actually the proper space interval.

This provides a physical explanation for length contraction as a manifestation of the reality of a particle's extended worldline, where the cross-section measured by an observer moving relative to it (*i.e.* at an oblique line in the *space−ct* diagram), creates the difference in perceived length between a body in its rest frame and a frame in movement, as seen in

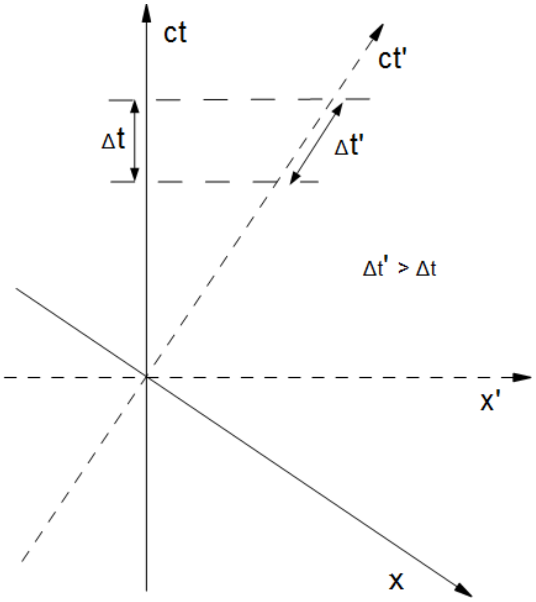

Fig. 1: Physical explanation of time dilation in a Loedel *space−ct* diagram

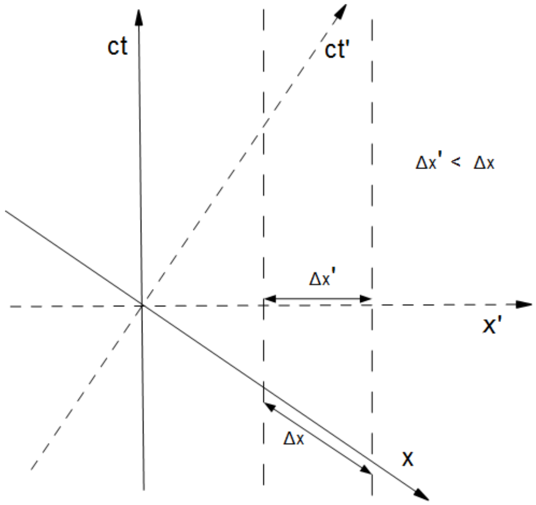

Fig. 2: Physical explanation of length contraction in a Loedel *space−ct* diagram

Fig. 2. It is important to understand that space itself is perceived to be contracted, not just objects in space. As seen in *STCED* [18], objects are not independent of spacetime, but are themselves deformations of spacetime, and are as such perceived to be contracted as space itself is. In actual practice, this phenomenon should be called *space contraction*, to avoid confusion, and demonstrate the complementary nature of time dilation and space contraction.

Thus we see that apparent time dilation and space contraction are perfectly valid physical results of Special Relativity, and there is nothing anomalous about them. Proper consideration of these phenomena eliminates the so-called paradoxes of Special Relativity as demonstrated by various authors, see for example [12, 14, 17]. We now explore the question of relativistic mass, which we first considered in section 2, in light of these considerations.

4 Relativistic mass as an effective mass

In this section, we show that the inertial mass of a body is its proper mass while the relativistic mass m^* is in effect an effective mass or, as Dixon [6] refers to it, an apparent mass. An effective mass is often introduced in dynamic equations in various fields of physics. An effective mass is not an actual mass – it represents a quantity of energy that behaves in dynamic equations similar to a mass. Using the effective mass, we can write the energy E as the sum of the proper mass and the kinetic energy K of the body, which is typically written as

$$E = m^* c^2 = m_0\, c^2 + K \tag{5}$$

to give

$$K = (\gamma - 1)\, m_0\, c^2 \,. \tag{6}$$

In reality, the energy relation in special relativity is quadratic, given by

$$E^2 = m_0^2\, c^4 + p^2 c^2 \,, \tag{7}$$

where p is the momentum. Making use of the effective mass (1) allows us to obtain a linear expression from (7), starting from

$$m^{*2} c^4 = \gamma^2 m_0^2\, c^4 = m_0^2\, c^4 + p^2 c^2 \,, \tag{8}$$

which becomes

$$pc = \sqrt{\gamma^2 - 1}\, m_0\, c^2 \tag{9}$$

or

$$pc = \beta\gamma m_0\, c^2 = \frac{v}{c}\, \gamma m_0\, c^2 = \frac{v}{c}\, E \,. \tag{10}$$

Then

$$p = m^* v \,. \tag{11}$$

As [12, p. 112] shows, the γ factor corresponds to the derivative of time with respect to proper time, *i.e.*

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{1}{(1 - \beta^2)^{1/2}} = \gamma \,, \tag{12}$$

such that the velocity with respect to the proper time, u, is given by

$$u = \gamma v \,. \tag{13}$$

Hence using (13) in (11) yields the correct special relativistic relation

$$p = m_0\, u \,, \tag{14}$$

which again shows that m^* in (11) is an effective mass when dealing with dynamic equations in the local time t instead of the invariant proper time τ. It is easy to see that differentiating (14) with respect to proper time results in a force law that obeys Newton's law with the proper mass acting as the inertial mass.

Hence we find that relativistic mass results from dealing with mass in local time t in a frame of reference moving with respect to the object of interest, instead of the invariant proper time τ in the frame of reference at rest with the object, and, from that perspective, is an effect similar to space contraction and time dilation seen in section 3. We see that the rest-mass m_0 should really be referred to as the proper mass, to avoid any confusion about the invariant mass of a body.

Relativistic mass is not apparent as time dilation and space contraction are, but rather is a measure of energy that depends on the relative speed v between two frames of reference, and is not an intrinsic property of an object as there is no absolute frame of reference to measure an object's speed against. The relativistic mass energy $m^* c^2$ is actually the total energy of an object (proper mass plus kinetic energy) measured with respect to a given frame of reference and is not a mass *per se* as mass is a relativistic invariant, *i.e.* a four-dimensional scalar, while energy is the fourth component of a four-vector.

5 Relativistic mass and *STCED*

In *STCED*, the proper mass corresponds to the invariant longitudinal volume dilatation given by [19, p. 32]

$$\rho c^2 = 4\kappa_0\, \varepsilon \tag{15}$$

which is equivalent to the inertial mass. The constant κ_0 is the spacetime bulk modulus and ε is the spacetime volume dilatation. Clearly, the longitudinal volume dilatation does not increase with velocity as it is an invariant. The result (14) is as expected from *STCED*.

For a spacetime volume element, the apparent space contraction in the direction of motion will be cancelled out by the apparent time dilation, *i.e.* the γ factors will cancel out. Thus the volume dilatation ε and the proper mass density ρ of (15) remain unchanged from the perspective of all frames of reference.

The only quantity that is impacted by the observer's frame of reference is the kinetic energy K or alternatively the quantity pc. In the frame of reference at rest with the object (which we can call the proper frame of reference), the kinetic energy $K = 0$ as seen from (6), while $pc = 0$ as seen from (9). The relativistic mass of an object is an effective mass defined to correspond to the total energy of an object (invariant proper mass plus kinetic energy) as observed from the perspective of another frame of reference. It does not represent an increase in the proper mass of an object, which as we have seen in section 4, corresponds to the inertial mass of the object.

6 Discussion and conclusion

In this paper, we have revisited the question of relativistic mass to clarify the meaning of this concept within special relativity. We have also considered time dilation and length contraction in more detail to help clarify the concept of relativistic mass. We have seen that "length contraction" is a misnomer and that it should really be named "space contraction" to avoid confusion, and demonstrate the complementary nature of time dilation and space contraction.

We have seen that relativistic mass is dependent on the difference in velocity v between an object's proper frame of reference that is at rest with the object and the frame of reference from which it is observed. We showed that the inertial mass of a body is its proper mass while the relativistic mass m^* is in effect an effective mass. We showed that relativistic mass results from dealing with dynamic equations in local time t in a frame of reference moving with respect to the object of interest, instead of the invariant proper time τ in the frame of reference at rest with the object. The results obtained are in agreement with the Elastodynamics of the Spacetime Continuum.

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