On the Classical Scaling of Quantum Entanglement

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In this paper, we consider the implications of the classical scaling of quantum entanglement observed experimentally. The probability of preserving entanglements over classical scales and preventing the entanglement from collapsing due to physical interactions is exceedingly small, indicating a fragile entanglement process. We propose a physically robust entanglement process that persists to classical scales as observed. We use a formulation of quantum mechanics that gives precedence to the physical rather than the mathematical aspects of the theory and its transition to the classical domain, using a physical interpretation instead of the literal interpretation of the Hilbert space of the standard formalism. We clarify the difference between separable (product) and non-separable (entangled) states, and the local realism nature of the product states which obey Bell's inequality compared to the non-local nature of the entangled states which violate Bell's inequality. We note that the truly quantum mechanical processes such as the double-slit interference pattern, potential barrier tunneling, and in particular the entanglement process as we show in this paper, depend on the quantum mechanical phenomenon of wave-particle duality. In entanglement experiments, the quantum mechanical results obtained are from the *wave* aspect of the wave-particle quantum object (q-object), just like the interference pattern in double-slit experiments, not the particle aspect of the q-object which is currently unknowingly assumed. The wave aspect of the q-object gives rise to the non-local behaviour as would be expected from the quantum mechanical calculations, while the particle aspect exhibits local causal behaviour. This explains why the entanglement process is robust: the wave-particle q-objects of entangled states have definite physical characteristics at emission time and are free of fragile evanescent properties. In addition, we conclude that "spooky action at a distance" (SAAD) is not required.

1 Introduction

Quantum entanglement is a quantum mechanical property of a composite quantum system consisting of two or more subsystems (such as particles), describing a situation where a quantum subsystem is linked to another via a specific process leading to correlations between observable physical properties of the subsystems. The two-particle *spin-singlet* state

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle\Big) \tag{1}$$

is an example of state entanglement in bipartite systems [1, p. 19].

Schrödinger first introduced the term *entangled state* to describe the non-separable pure states of quantum systems [2], [1, p. 17]. Consider for example the emission of two photons of opposite polarization from a given process, such as the stimulated emission of polarization-entangled photons (see for example [3, 4]). The emitted photons are then conceived of as "entangled" pure states. The system is described by the wavefunction [4]

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\circlearrowright_1\rangle|\circlearrowleft_2\rangle + |\circlearrowleft_1\rangle|\circlearrowright_2\rangle\Big) \tag{2}$$

where $\circlearrowright_i$ and $\circlearrowleft_i$ represent the right-hand and left-hand circularly polarized photons for $i = 1$ or 2. This wavefunction represents what we know of the entangled system, or alternatively represents our lack of knowledge of the specific properties of each photon that is emitted. All we know is that if one emitted photon is right-hand circularly polarized, then the other will be left-hand circularly polarized, and vice versa. Eq. (2) is a statement of this situation.

The predominant interpretation (the orthodox viewpoint [5]) is that the wavefunction (2) represents a physical description of the emitted photons in an unresolved evanescent state, and that once a measurement is performed on one of them, the wavefunction collapses, the measured photon's actual properties are then known and an instantaneous propagation of that information is perceived by the other photon so that it can assume the complementary properties required by the process – "spooky action at a distance" (SAAD) as Einstein called it, a process that some physicists like to think of as quantum magic, an approach that speaks more of metaphysics than physics. The reasons for the acceptance of this description will be considered in greater detail in Section 5.

Over the past decades, experiments have been devised to extend the range of quantum entanglements, to the point where classical scales have been achieved. This includes both the size of entangled objects (*e.g.* [6–10]) and the distances over which entanglement has been maintained (*e.g.* [11, 12]).

These are particularly stunning results as any interaction of one of the entangled components with its environment will collapse the entanglement. The probability of preventing such interactions and preserving entanglements over classical sizes and distances is exceedingly small. As noted by Jaeger [1, p. 20] "Indeed, *pure* such states of two-particle systems are exceptional rather than typical in the world; typically, a system very soon interacts with a number of other systems, so that, even if it were prepared in a pure state, it is typically described by a mixed state".

The probability that a photon can travel a distance x without interaction is given by [13] [14, Section 3.3.1] [15, p 304]

$$P_{no-int}(x) = \exp(-n_p\, \sigma\, x) \tag{3}$$

where n_p is the particle number density and σ is the total photon interaction cross-section including absorption and scattering. For propagation of photons in the atmosphere, $n_p \sim 2.5 \times 10^{25}\,\mathrm{m}^{-3}$ [16] and $\sigma \sim 180\,\mathrm{barn/molecule} \equiv 1.8 \times 10^{-26}\,\mathrm{m}^2/\mathrm{molecule}$ [17]. Using these values in (3), the no-interaction probability becomes

$$P_{no-int}(x) = e^{-0.45\,x} \tag{4}$$

where x is in meters. We see that for classical distances x, the probability $P_{no-int}(x)$ increasingly becomes very small. For example, $P_{no-int}(1\,\mathrm{m}) = 0.64$, $P_{no-int}(10\,\mathrm{m}) = 0.011$, $P_{no-int}(100\,\mathrm{m}) = 2.9 \times 10^{-20}$, $P_{no-int}(1\,\mathrm{km}) = 3.6 \times 10^{-196}$. For the value of 143 km of [11, 12] the probability that a photon can travel such a distance without interaction is astronomically small.

Hence the probability of preserving entanglements over classical sizes and distances and preventing the entanglement from collapsing due to physical interactions is exceedingly small. The question has to be raised: in light of these successful classical-scale experiments, are we currently misunderstanding the quantum entanglement process such that instead of a fragile entanglement situation as the above considerations indicate, we can derive a quantum entanglement process that leads to a physically robust entanglement situation that persists to classical scales as observed?

2 Quantum entanglement questions

Questions have been raised concerning entanglement and its extension to the classical (or macro) domain [18]. There is no doubt that some processes generate particle or photon pairs that have a definite relationship (correlation) between them (which are referred to as being entangled) and these relationships are confirmed experimentally. At stake here is the interpretation of the quantum entanglement process, and the impact of the understanding of this process on the development and technological applications of this quantum mechanical process – a misinterpretation can lead to considerations that are not physically realistic.

Questions have also been raised on the limited applicability of Bell's inequality [19–21], based on the assumptions used in its derivation. Bell [22] uses a single continuous parameter λ described by a probability distribution $\rho(\lambda)$: the basic limitation of this approach is that it imposes a quantum mechanical calculation approach on the analysis. Bell's derivation is only applicable to a specific class of hidden variable theories that can be represented by his starting equation and assumptions, which Jaynes [20] refers to as Bell theories. Some hidden variable theories don't need to satisfy Bell's starting equation to reproduce quantum mechanical results, as evidenced by Bohmian mechanics [23]. Bell's inequality is thus found to apply to a limited set of circumstances and situations, not to every quantum system. Selleri [24] provides a comprehensive review of the proofs of Bell's inequality.

Actual experimental demonstration of entanglement is a challenge. Entanglement experiments detect both entangled components within the same time window (see Subsection 5.3), so there is no way to confirm the presence or absence of SAAD – it is assumed to be present purely based on the predominant interpretation discussed in Section 1. Zhao [19] has proposed various experiments to clarify the physical properties of entanglement, including one to determine if the collapse of the entangled wavefunction due to the measurement of one component causes the transformation of the other component due to SAAD as is supposed in the orthodox interpretation. No reports of these experiments having been performed have surfaced – their execution should be given a high priority to help us better understand the phenomenon of entanglement.

3 Literal or physical interpretation?

To be able to answer the question posed at the end of Section 1 on a physically robust entanglement process, we need to have a better understanding of the physical description of quantum mechanics and of its transition to the classical domain. The orthodox view in the standard formalism of quantum mechanics is done via entanglement, wavefunction collapse and decoherence [25]. This is a literal interpretation of the Hilbert space mathematical theory of quantum mechanics developed by von Neumann and Dirac [26, 27]. However, as noted by Home and Whitaker [15, see p. 309], "[t]o conclude, there are aspects of classical reality pertaining to the macrophysical world that cannot be made consistent with quantum theory in any limit, at least using the standard formalism and decoherence models."

This thus leads us to consider other approaches to understand this problem. There are other interpretations of quantum mechanics which satisfy its principles – the book by Home [14] provides an excellent exposition of the conceptual foundations of quantum physics. As is well-known [28], the various formulations of quantum mechanics provide the same results (Schrödinger wave equation, Heisenberg matrix formulation, Dirac standard formalism, Feynman path integral, Bohm quantum potential among others) – the differences between them lie in the insights that these different formulations can provide. To understand the process under discussion, what is required is a physical interpretation based on a formulation of quantum mechanics that gives precedence to the physical rather than the mathematical aspects of the theory, and of its transition to the classical domain.

A physical theory of quantum mechanics which offers a logical transition into classical physics was first developed before it was displaced by the preferred standard formalism. This initial theory was instrumental in the development of quantum mechanics. Here we briefly recap this approach.

In classical mechanics [29], the phase space description of a system is given in terms of generalized coordinates $q = \{q_i\,;\ i = 1, 2, \cdots, N\}$ and canonical momenta $p = \{p_i\,;\ i = 1, 2, \cdots, N\}$ and its time evolution is described in terms of its Hamiltonian $H(q, p)$ using Hamilton's equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i}\,, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}\,. \tag{5}$$

The Lagrangian of the system determines its dynamics in configuration space in terms of the coordinates $\{q_i\}$ through the Euler-Lagrange equations

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0\,,\ i = 1, 2, \cdots, N\,. \tag{6}$$

If a statistical description of the system is desired, the state of the system is described in terms of a probability function $P(q, p)$ defined on the phase space, and its time evolution is given by

$$\frac{dP}{dt} = \{P, H\} + \frac{\partial P}{\partial t}\,, \tag{7}$$

where the Poisson bracket $\{P, H\}$ is given by

$$\{P, H\} = \sum_i \left(\frac{\partial P}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial H}{\partial q_i}\frac{\partial P}{\partial p_i}\right). \tag{8}$$

The quantum mechanical description of the system derived from the foregoing considerations sees the dynamical variables (q, p) now interpreted as operators $(\hat{q}, \hat{p})$ acting on complex wavefunctions $\psi(q)$ generating observables and satisfying the commutation relation

$$[\hat{q}_i, \hat{p}_j] = i\hbar\, \delta_{ij}\,, \tag{9}$$

where $\hbar$ is Planck's reduced constant. This transition from a classical to a quantum mechanical description, known as *canonical quantization*, is effected (done) by the replacement of classical variables by quantum operators according to

$$q_i \rightarrow \hat{q}_i\,, \quad p_i \rightarrow \hat{p}_i \tag{10}$$

and (classical) Poisson brackets by (quantum) commutators according to

$$\{A, B\} \rightarrow \frac{1}{i\hbar}\,[\hat{A}, \hat{B}]\,. \tag{11}$$

The close relation between the classical and quantum dynamical equations is evident in the similarity between the classical equation of motion (7) and the quantum equation of motion as derived by Heisenberg,

$$\frac{d}{dt}\langle A\rangle = \frac{1}{i\hbar}\,\left\langle[\hat{A}, \hat{H}]\right\rangle + \left\langle\frac{\partial A}{\partial t}\right\rangle. \tag{12}$$

This result is a manifestation of Ehrenfest's theorem [30, see pp. 389–394] which holds that quantum mechanical expectation values $\langle A\rangle$ obey the classical equations of motion. This similarity points to the relation between the classical probability functions defined on the (q, p) phase space and the quantum mechanical expectation values obtained from the $(\hat{q}, \hat{p})$ operators acting on the complex wavefunctions $\psi(q)$ representing our knowledge of the system, which in the end obey the classical equations of motion.

This approach provides a physical interpretation that can be used to better understand the classical scaling of quantum entanglement. One of the characteristics of the above considerations is the physical reality of the underlying quantum mechanical system as it evolves into a classical system. In the following section, we consider the nature of quantum states as this has an impact on the robustness of entangled states.

4 The nature of quantum states

Jaeger [1, pp. 19–22] clearly communicates the importance of understanding the difference between separable (product) and non-separable (entangled) states. Over the past quarter century, the definition of entanglement has been extended, from information theory, to include mixed states that are separable when given as combination of products of subsystem states. Separable subsystem states are entirely uncorrelated (not entangled), while the entangled mixed states are the inseparable states – however, "[t]he problem of determining whether or not a given state of a composite system is entangled is known as the *separability problem*." [1, p. 21]. These entangled mixed states tend to somewhat muddle the entanglement water.

When considering separable (product) states, as noted by Jaeger [1, p. 21], "...the outcomes of local measurements on any separable state can be simulated by a local hidden-variables theory, that is, the behavior of systems described by such states can be accounted for using common-cause explanations". In other words, *separable* states can have definite physical properties when they are prepared.

It is important to note that Bell's inequality is violated only by entangled (non-separable) states. As noted by Jaeger [1, p. 22], "[t]he quantum states in which correlations between [components] A and B can violate a Bell-type inequality are called *Bell correlated*, or *EPR correlated*. If a bipartite *pure* state is entangled, then it is Bell correlated with certainty, as was first pointed out by Sandu Popescu and Daniel Rohrlich [31] and by Nicolas Gisin in the early 1990s [32].

However, no simple logical relation between entanglement and Bell correlation holds for the mixed entangled states".

Home [14, pp. 203–209] also makes the point. He concludes "an arbitrary mixture of factorable or product state vectors always satisfies Bell's inequality" as first shown by [33, 34], while " [f]or any given nonfactorable state vector of correlated quantum systems it is always possible to choose observables so that Bell's inequality is violated by quantum mechanical predictions." [14, pp. 205, 208] which was first demonstrated by [32] as seen previously.

Hence we have two different types of quantum states depending on whether they are product (separable) or entangled (non-separable) states. Separable states are consistent with local realism – they can be physical and local, while entangled states are not consistent with local realism, based on Bell's inequality. The normal reaction would be that there should be one consistent behaviour across all states, that the entangled states' behaviour trumps the separable states' behaviour, and hence quantum states are not consistent with local realism.

However, as seen in Section 2, questions have been raised about Bell's inequality, and this difference in behaviour between separable and entangled states may indicate that there is a problem with our understanding of Bell's inequality and of entanglement in general. We explore this question in greater details in the next section, and in doing so, show that we can in fact derive a robust entanglement process as observed in the classical scaling of quantum entanglement.

5 A robust entanglement process

The considerations of Section 3 reinforce the underlying physical building blocks of quantum mechanics: the superposition principle, Heisenberg's uncertainty principle and wave-particle duality. These are crucial to physically understand the entanglement process and demonstrate why it is a robust process. While the superposition property results from the linear wave equations used in the theory and Heisenberg's uncertainty principle results from the fact that quantum mechanical canonically conjugate dynamical variables are Fourier transform pairs of variables [35], wave-particle duality is a purely quantum mechanical property and is undoubtedly the most important of these. The truly quantum mechanical processes such as the double-slit interference pattern, potential barrier tunneling, and in particular the entanglement process as we will see in this section, depend on the quantum mechanical phenomenon of wave-particle duality. It is critical to analyze quantum phenomena in terms of wave-particle duality to fully understand them.

5.1 Non-existence of hidden-variables?

Home [14] does an extensive review of all proofs of the non-existence of hidden-variable theories in quantum mechanics and concludes "[h]aving established that contrary to folklore, no a priori compelling argument excludes the possibility of contextual hidden variable theories, the entire enterprise of developing a more complete description of quantum phenomena beyond the ambit of the standard interpretation becomes logically legitimate", and provides a reference to an example: "A pedagogically instructive model example of how a contextual hidden variable model can reproduce the standard quantum mechanical results is discussed by [36], who show in detail how such a model can provide an objectively real treatment of decaying, oscillating, and regenerating kaons" [14, pp. 195–196]. A contextual hidden variable model is one "in which the value obtained by a measurement is a function of the premeasurement value as well as the measurement context." [14, p. 37].

In addition, the basic deficiency of hidden-variable non-existence proofs is that they are derived within the context of quantum mechanics. By its very nature, quantum mechanics is a probabilistic theory – so it is not surprising that such "proofs" find that deterministic results cannot be derived from quantum mechanics. The reader is referred to [21] for an example of this approach in the assumptions used by Bell in the derivation of his inequality, which leads to the conclusion that "it is not surprising that Bell's inequality is not satisfied in systems that obey quantum mechanics".

It is important to note that the label "hidden-variable theories" is attached indiscriminately to more complete theories of quantum mechanics. However, as in the case of Bohmian mechanics, a deterministic quantum physics theory does not need to include hidden variables. The proper path to such a theory is to start outside of quantum mechanics, derive a deterministic microscopic theory, and show that quantum mechanics can be derived from it – see [44] for an example of this approach.

Home [14] continues "[t]here are strong physical grounds for suspecting that the standard framework (formalism and interpretation) of quantum mechanics is fundamentally inadequate, though its empirical success to date is unquestionably impressive" [14, p. 37]. Home identifies the following aspects of quantum mechanics that are not well understood in the standard framework: the quantum measurement paradox, the classic limit of quantum mechanics, nonlocality of quantum mechanics arising from entanglement, and wave-particle duality [14, pp. 37-38]. These are the very factors at play in the robustness of the entanglement process as discussed in this paper.

5.2 Wave-particle q-objects

Entanglement experiments compare the behaviour of classical particles with quantum mechanical results that are unknowingly assumed to represent the particle aspect of the wave-particle quantum object (which for brevity we refer to as a "q-object"). It is important to realize that a q-object does not behave as a classical object due to its explicit wave-particle nature. For the wave aspect of a macroscopic object, its de Broglie wavelength is extremely small and its effect is negligible – however, in the quantum mechanical domain the impact of the wave-particle nature of the q-object becomes significant as observed in quantum physics. It is interesting to note that the impact of wave-particle duality has been observed at mesoscopic scales as reported in [6]. Thus a q-object is an object where the effect of wave-particle duality cannot be neglected.

In entanglement experiments, the quantum mechanical results obtained are from the *wave* aspect of the wave-particle q-object, just like the interference pattern in double-slit experiments. Hence, the results obtained in Bell experiments [38] and other entanglement experiments devised since then are the quantum mechanical results of the wave aspect of the wave-particle q-objects which are different from the particle results, again as seen in double-slit experiments (classical double-particle pattern versus quantum mechanical wave interference pattern). Similarly in Hardy experiments [39], the non-zero probability $P(A_1, B_1)$ [40] obtained in contradistinction to the local realist probability of zero is due to the *wave* aspect of the wave-particle q-object.

Wave-particle duality is still somewhat of a mystery in quantum mechanics. It is still understood mostly in terms of Bohr's principle of wave-particle complementarity which holds that the wave aspect and the particle aspect of an object are complementary aspects of a quantum object [14, see Chapter 5]. However, wave-particle duality arises naturally in the theory of Spacetime Continuum Elastodynamics (*STCED*) [57, 58] which is briefly covered in the Appendix and is considered in greater detail in [44]. This model provides a natural explanation for wave-particle duality, where an object, represented as a spacetime deformation, is composed of transverse and longitudinal modes, with the transverse mode corresponding to the wave aspects of the deformation and the longitudinal mode corresponding to the particle aspects of the deformation.

A wave-particle q-object is thus a hybrid object consisting of both wave and particle aspects which manifest themselves differently in experiments, depending on the type of measurement. We examine the experiments of Aspect *et al.* [41–43] using single-photon states covered in Home [14, Section 5.4] to demonstrate how they can be fully understood in terms of *STCED* wave-particle duality.

In the "light pulses on a beam splitter" experiment (Home's Fig. 5.2), for a pulsed photodiode light pulse, the wave aspect is expected to apply from the *STCED* wave-particle model – indeed, as Home comments "[t]he striking feature is that even under this apparently quantum condition, light pulses arriving at the beam splitter continued to behave as classical waves, and the inequality $[P_C \geq P_T P_R]$ was never observed to be violated" [14, p. 288], where P_T is the probability that a single count is transmitted, P_R is the probability that a single count is reflected, and P_C is the probability of a coincidence for that single count.

For a source of single photon pulses from an excited atom transition, using the same experimental setup, the particle aspect is expected to apply from the *STCED* wave-particle model – indeed, "a clear-cut violation of the inequality $[P_C \geq P_T P_R]$" was observed. "This confirmed single particle behavior of the single-photon states." [14, p. 288].

The experiment was then modified as per Home's Fig. 5.3 by removing the detectors on either side of the beam splitter and recombining the two beams using mirrors and a second beam splitter. Using the source of single photon pulses from an excited atom transition as previously, this time the wave aspect is expected to apply from the *STCED* wave-particle model as it is being treated as a wave (recombining the two beams) – indeed, the experiment "showed interference effects dependent on the difference in path lengths along two possible routes of single-photon pulses." [14, p. 288].

This provides experimental confirmation of the *STCED* wave-particle model where the wave-particle q-object consists of both wave and particle aspects which manifest themselves differently depending on the type of measurement. The behaviour is physical and logical. In addition, nothing precludes the wave-particle q-object from having the full physical properties encoded in the q-object. The results obtained in the case of non-rotated detectors are in agreement with local results that would be obtained classically, because there are no specific quantum effects coming out of the quantum mechanical calculations in this case.

This indicates that the entangled q-objects are emitted with deterministic physical properties. The wave aspect gives rise to the non-local behaviour (within causality requirements due to the particle aspect of the q-object) as would be expected from the quantum mechanical calculations, while the particle aspect exhibits local causal behaviour [44]. This explains why the entanglement process is robust: the wave-particle q-objects of entangled states have definite physical, not evanescent, characteristics at emission time.

5.3 Physical approach

This leads us to consider a physical approach which posits that the photons (for example), as wave-particle q-objects, are emitted with specific properties, but that due to our lack of knowledge of their detailed characteristics, can only be probabilistically characterized with the wavefunction ψ as a combination of the possible states and their probabilities (the realistic viewpoint [5]). Once a measurement is performed on one of the photons, its properties are resolved, thereby increasing our knowledge of the system, and allowing us to specify the properties of the other photon – a simple physical understanding of the process [21]. Such a process can easily scale to classical objects and distances, and is undeniably very robust as the q-objects' properties are determined at emission time, not evanescent depending either on an experimenter's whim or thought process, or on not having an interaction that would destroy the entanglement on its way to measurement resolution. The classical-scale experiments considered previously are then seen to be a confirmation of this approach.

The wavefunction is thus seen to be a probabilistic description of our (limited) *knowledge* of a quantum mechanical system, not a complete physical description of the system, with this probability being proportional to the *intensity* of the wavefunction as seen in [44]. This explains the laws of quantum probability [45,46]. We note the same behaviour for electromagnetic radiation, where the intensity is proportional to the energy density of the field, which can be converted to a probability by normalization, as seen in [44].

As a result of the measurement process, the original wavefunction description is superceded (the so-called collapse of the wavefunction) and is replaced by a more accurate wavefunction description of the quantum mechanical system that takes into account the results of the measurement process. As [37] puts it, "When a detector clicks the wavefunction does not 'collapse' from all over space to a point, it is simply that only part of it is now relevant.". It is important to note that this measurement process is effected (done) by the interaction of the quantum mechanical system with an outside agency, whether it is a measurement apparatus or an interaction with another quantum mechanical system.

This is a simple logical description of the physical process that does not require metaphysical "spooky action at a distance" explanations and, by the principle of Occam's razor, is a superior explanation of the entanglement process. It should be noted that the imaginary actors "Bob" and "Alice" which are used in the explanation of entanglement and SAAD, even though the explanation is presented as a sequential series of events, are both aware of the same experimental information within the same time window, as mentioned in Section 2, and hence fully satisfy Jaynes' analysis of entanglement experiments as discussed in [20, 21].

As Home points out, "[c]ontrary to a widely held misconception, we stress that no experiment probing quantum locality has yet tested quantum correlations measured across spacelike separation unambiguously." [14, p. 233]. In photon polarization correlation experiments [38], "[t]he claim of spacelike separation is usually based on ensuring that a photon on one side reaching a photomultiplier detector is spacelike separated from its partner passing the polarization analyzer on the other side." However, a typical photomultiplier detector requires about 30 ns for a current pulse to be generated following the arrival of a photon, which provides a different spacelike separation than that obtained from the resolution time of a photomultiplier which is usually of order 1 ns [14, p. 233].

It should be noted that the model proposed in this paper is independent of these so-called "loopholes". They are mentioned to indicate the difficulty of performing such experiments which raises cautionary notes on the concomitant dangers of wishful thinking and unrecognized assumptions, limitations and interpretation of the results.

5.4 Evidence for SAAD?

So why introduce a mysterious agent, "spooky action at a distance", when none is required? As we asked in Section 1, what prompts the acceptance of this description as part of the orthodox interpretation? The reason is that SADD is believed to be supported by the experimental evidence. However, the aforementioned considerations and the analysis of Jaynes [20,21,47] show that the experimental evidence can be explained without resorting to metaphysics, that the problem results from the assumption that a conditional probability represents a physical influence instead of the physically-correct logical inference that it is.

As Home and Whitaker write [15, p. 238],

> In one out of four cases, Alice is lucky with her measurement, and Bob's particle immediately becomes an identical replica of Alice's original. Then it might seem as if information has traveled instantly from Alice to Bob. Yet this strange feature cannot be used to send *usable information instantaneously*, because Bob has *no* way of knowing that his particle is already an identical replica. Only when he ***learns*** the result of Alice's Bell-state measurement, which is transmitted to him via classical means, can he exploit the information in the teleported quantum state.

where the emphasis is in the original text and we have in addition highlighted the word "learns".

In other words, what is believed to be "spooky action at a distance" is actually the experimenters' knowledge of the system suddenly increasing as a result of the measurement process, and the experimenters being in a position to logically infer the properties of the distant component, which is confirmed in the measurement performed on the distant component. In actual practice, in entanglement experiments, both measurements are done in the same time window (see Sections 2 and 5.3).

There is also a certain intellectual inertia at play. As Bell [48] commented, "Why is the pilot wave picture [Bohm's] ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?" All very good questions.

6 Quantum information causality

The emerging concept of *information causality* [49–51] is an attempt to preserve causality based on the underlying premise that it is information that is the core element in the analysis of the entanglement process. The approach followed is to impose this concept as a principle of nature to avoid the special relativistic causality problems raised by SAAD. This concept unwittingly reflects Jaynes' analysis of entanglement experiments in that it focuses on information – however, Jaynes' analysis [20, 47] already accomplishes this without having to introduce an additional constraint in the guise of a new causality principle, and in so doing, also eliminates the need for SAAD.

7 Weak quantum measurements

Weak quantum measurements [52–56] is another emerging concept in quantum mechanics that has an impact on the understanding of the entanglement process. What is interesting with this approach is that it is possible to make minimal-interacting measurements, which leaves the collapse of the wavefunction in the literal interpretation of the mathematical standard formalism of quantum mechanics in a quandary: how can any measurement be done without collapsing the wavefunction?

The accepted explanation [54] is that the quantum state is not collapsed into eigenvectors, but instead, by a weak coupling of the measurement device and the system, is biased by a small angle such that the measurement device shows a superposition of several eigenvalues. The current status is summarized as follows: "weak measurement theory presents a plethora of strange quantum phenomena, not yet completely understood." [54]. There is no doubt that even a weak interaction measurement will have an impact on the system, and this approach, certainly experimentally valid, puts the wavefunction collapse of the literal interpretation of quantum mechanics into question.

The proposal of weakly interacting measurements was also introduced in [35] in the context of the application of the Nyquist-Shannon Sampling Theorem to quantum measurements. The author showed that Brillouin zones in Solid State Physics are a manifestation of the Nyquist-Shannon Sampling Theorem at the quantum level, where the translational symmetry of atoms in a solid resulting from the regular lattice spacing, is equivalent to an effective sampling of the atoms of the solid giving rise to the Brillouin zones. This raised the possibility of investigating new experimental conditions leading to new measurements previously considered unreachable, a possibility that is also considered possible in the literature on weak quantum measurements.

8 Discussion and conclusion

In this paper, we have considered the classical scaling of quantum entanglement. This implies a physically robust entanglement process, contrary to the fragile entanglement process that the standard formalism interpretation implies given that the probability of preserving entanglements over classical sizes and distances and preventing the entanglement from collapsing due to physical interactions is exceedingly small.

Actual experimental demonstration of entanglement, other than testing the Bell inequality, is a challenge. Entanglement experiments detect both entangled components within the same time window, so there is no way to confirm the presence or absence of "spooky action at a distance" (SAAD) which is assumed to be present based on the standard formalism interpretation.

To better understand the entanglement process and determine a robust entanglement process, we have considered a physical interpretation based on a formulation of quantum mechanics that gives precedence to the physical rather than the mathematical aspects of the theory used in the literal interpretation of the Hilbert space formulation.

We have considered the transition from a classical to a quantum mechanical description, known as *canonical quantization*, which is effected (done) by the replacement of classical variables by quantum operators, and have noted that one obtains closely related classical and quantum (Heisenberg) equations of motion. This result is a manifestation of Ehrenfest's theorem which holds that quantum mechanical expectation values obey the classical equations of motion.

We have considered the difference between separable (product) and non-separable (entangled) states. Mixtures of product (separable) states always satisfy Bell's inequality *i.e.* separable states can have definite physical properties when they are prepared. Bell's inequality fails only for entangled (non-separable) states. Hence separable states are consistent with local realism – they can be physical and local, while entangled states are not consistent with local realism, based on their violation of Bell's inequality.

We have seen that these considerations reinforce the underlying physical building blocks of quantum mechanics: the superposition principle, Heisenberg's uncertainty principle and wave-particle duality which is the most important of these. The truly quantum mechanical processes such as the double-slit interference pattern, potential barrier tunneling, and in particular the entanglement process as we have seen in this paper, depend on the quantum mechanical phenomenon of wave-particle duality. It is thus critical to analyze quantum phenomena in terms of wave-particle duality to fully understand them.

We have noted Home's [14] conclusion reached after an extensive review of all proofs of the non-existence of hidden-variable theories, that "no a priori compelling argument excludes the possibility of contextual hidden variable theories", giving legitimacy to the development of a more complete description of quantum phenomena beyond the standard interpretation. He further identifies the aspects of quantum mechanics that are not well understood in the standard framework: the quantum measurement paradox, the classic limit of quantum mechanics, nonlocality of quantum mechanics arising from entanglement, and wave-particle duality, which are the very factors at play in the robustness of the entanglement process as discussed in this paper.

We have noted that in entanglement experiments, the quantum mechanical results obtained are from the *wave* aspect of the wave-particle quantum object (which for brevity we refer to as a "q-object"), just like the interference pattern in double-slit experiments. A q-object is an object where the effect of wave-particle duality cannot be neglected.

Hence, Bell's inequality is violated in the quantum mechanical problem, that is the wave aspect of the wave-particle q-object, which is different from the particle results, as seen in double-slit experiments (particle versus wave patterns). However, nothing precludes the wave-particle q-object from having the full physical properties encoded in the q-object when the entangled q-objects are emitted. The wave aspect then gives rise to the non-local behaviour (within causality requirements due to the particle aspect of the q-object) as would be expected from the quantum mechanical calculations, while the particle aspect exhibits local causal behaviour. This explains why the entanglement process is robust: the wave-particle q-objects of entangled states have definite physical characteristics at emission time.

This has lead us to consider a physical approach which posits that the photons (for example), as wave-particle q-objects, are emitted with specific properties, but that due to our lack of knowledge of their detailed characteristics, can only be probabilistically characterized with the wavefunction ψ as a combination of the possible states and their probabilities (the realistic viewpoint). Performing a measurement on one of the photons resolves its properties which allows us to specify the properties of the other photon – a simple physical understanding of the entanglement process. Such a process can easily scale to classical objects and distances, and is undeniably very robust as the q-objects' properties are determined at emission time, not evanescent as in the standard formalism. The classical-scale experiments considered previously are then seen to be a confirmation of this approach.

We have also considered the emerging concept of *information causality* which is an attempt to preserve causality based on the underlying premise that it is information that is the core element in the analysis of the entanglement process, which is correct. However, Jaynes' analysis [20, 47] already accomplishes this without having to introduce an additional constraint in the guise of a new causality principle, and in so doing, also eliminates the need for SAAD.

We have also considered weak quantum measurements which is another emerging concept in quantum mechanics. There is no doubt that even a weak quantum measurement will have an impact on the system, and this approach, certainly experimentally valid, puts the wavefunction collapse of the literal interpretation of quantum mechanics into question.

It should be noted that quantum cryptography and quantum computing are then seen to depend on the *wave* aspect of the wave-particle q-object. This fundamental understanding should help accelerate the progress of these new development programs.

The resolution of the robustness of the entanglement process in classical scale quantum entanglement experiments is thus achieved within the wave-particle q-object explanation of the process in which entangled state q-objects have definite physical characteristics at emission time. Strong evidence has been provided to support this proposal.

The design of experiments to provide experimental evidence requires that experimentalists shift the paradigm used to test quantum theories. Currently experiments are designed to try to prove the applicability of quantum mechanics to entangled states by verifying various inequalities such as Bell's. The experiments suggested by Zhao [19] try to clarify the physical properties of quantum entanglement and includes experimental tests of the locality of the measurements of Bell states, experimental tests of the constituents of Bell states, and experimental tests of determinism in quantum measurements. In addition, even though the entanglement experiments currently performed agree with the model proposed in this paper, specific experiments need to be performed to test the model under conditions that emphasize that quantum entanglement behaviour results from the *wave* aspect of the wave-particle q-objects.

Appendix: wave-particle duality in *STCED*

It should be noted that wave-particle duality is considered in greater detail in [44] within the theory of the Elastodynamics of the Spacetime Continuum (*STCED*) [57, 58]. As shown in *STCED*, energy propagates in the spacetime continuum as wave-like deformations which can be decomposed into *dilatations* and *distortions*. *Dilatations* involve an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation. On the other hand, *distortions* correspond to a change of shape of the spacetime continuum without a change in volume and are thus massless. Thus the deformations propagate in the continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements. This provides a natural explanation for wave-particle duality, with the transverse mode corresponding to the wave aspects of the deformation and the longitudinal mode corresponding to the particle aspects of the deformation.

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