Predictability Is Fundamental

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1 General concept of trajectory

In his relationship with Nature, the person might be active if he wants to get to some state of the world, and then he is looking for a means to reach this state. Although the content of the state is completely in his mind, he needs the prediction for his action to reach the desired. Typically, this is difficult in real life, and people act according to more or less uncertain hopes, past experience, beliefs etc. However, sometimes predictions might exist to recommend actions with the universally guarantied results – always and everywhere. Though infrequent, such predictions are therefore recommended to be looked for first of all, and our so valued technologies are based solely on these.

The related scheme of the world states must be able to formulate predictions in its own internal terms. If some state in the scheme is associated with the desired, so being the final for the person's purpose, the initial state, from which the action should start, must be defined in the internal terms of the scheme as well. Since the final state is not reached as yet, it should be set in the future with respect to the initial. If being in the initial state the person is guaranteed to reach the final, no prediction is needed. As the first order development, we might include in the scheme some intermediate state such that transitions from the initial state to this intermediate and from the intermediate to the final are both sure. Then the problem is reduced to finding this intermediate state. Only one such state might be there, because the existence of even one more would provide uncertainty as to which one to choose, so making the prediction incomplete.

Giving the number 0 to the initial state and 1 to the final, let us give $\frac{1}{2}$, say, to this intermediate (no metric is implied – just the order). In the same way we define next $\frac{1}{4}$ and $\frac{3}{4}$ states and so on. This procedure involves only rational numbers, so some infinite sequences of the states might not converge to a state with the rational number to become the initial for the further part of the sequence. Therefore all sequences, i.e., all real numbers are required for guaranteed predictions (Dedekind). In so doing, only order is important, and a state might correspond either to the rational or irrational number as well. Again, no state not belonging to this sequence can exist in the prediction of the steady transition from 0 to 1, otherwise the prediction becomes incomplete. In the Lagrange's version of mechanics, its basic least action principle reflects just this singleness.

Such state sequences are called trajectories, and we are ready now to approach the Newton's scheme, starting with the very condition of the universal predictability. It should be stressed that the scheme is only the necessary language for making universal predictions; it is supported by, though not coming from, our senses that connect us with Nature also in great many other respects.

2 Principles of the Newtonian mechanics

In this essay, I don't consider the post-Newton development of his ideas; even the contribution of Maxwell and Einstein will not be discussed here. My purpose is to understand whether or not the very scheme of mechanics elaborated by Newton is the only possible one. Upon working over many decades in experimental physics, I couldn't refrain from asking myself as to what if there is some other and more efficacious way to address Nature. To this end, I'm going to scrutinize the Newton's scheme in every respect.

Following the method of Descartes of representing geometrical figures with numbers and related equations, Newton has formulated his three 'Laws of Mechanics' in order to apply the similar procedure to physics, i.e., to describe also motion by means of Cartesian coordinates.

The first Newton's law introduces rectilinear and uniform trajectories as free from an external influence ("force"). However, this law is just a vicious circle. As Einstein mentioned in his "The Meaning of Relativity": "The weakness of the principle of inertia lies in this, that it involves an argument in a circle: a mass moves without acceleration if it is sufficiently far from other bodies; we know that it is sufficiently far from other bodies only by the fact that it moves without acceleration."

Aiming at numbering arbitrary motions, we have first of all to match abstract geometric images with real operations. Indeed, what does it mean "rectilinear" in Nature? How rectilinear a trajectory should be for the scheme still being suitable? How to make it sure that a line is straight? Suffices it to be described by linear equations in a reference frame formed as the Cartesian structure? But then, we have to recognize first that our reference frame itself is comprised of straight axes. The commonly accepted agreement suggests using some standard rigid rods. How rigid? Sometimes rigid might appear soft. This depends on the inter-atomic distances, but the concept of distance is still to be introduced using standard rods. (Circle!) We are to transport the rod as along the reference frame axes for marking them evenly, so also over the whole space with parallel shifts and rotations, being sure that it remains rigid. In so doing, we believe that no actions destroy these operations. The marks on the axes define Cartesian coordinates, which will further be used to define a scalar – squared "length" as the sum of the squared coordinate differences. Only then can we construct the full Cartesian structure using equal length rods to obtain the necessary symmetries of reference frames. (One more circle!) Also collimated light rays might be used, whenever diffraction (still depending on wavelength!) could be neglected, either solely to define linearity, or together with rods for parallelism and other symmetries. Being applied to measurements of motions, we inquire the relevance of these devices, since in fact this procedure has nothing to do with the motion in question. It might well happen that in the study of motion our artificial rods either add something of their own or hide something, so being suitable within only some limited scope of motions. We cannot refer here to great many successful technical applications as well as to the broad experimental support, since all these are carried out within the introduced in advance basic conceptions, so being relevant only within some narrow areas of the implied research.

Even more difficult questions spring up upon considering the time intervals measurement and its universal applicability to real motion. How do we know that the duration of one hour now is equal to that in the future (see, e.g., H.Weyl, "Space-Time-Matter")? How uniform free motion is to be for the scheme to remain suitable? Beginning with Zeno, Aristotle etc., philosophers were burdened by the mystery of time, and Newton himself attempted, in vain, to develop the concept of "genuine" time, that runs uniformly and is free from any influence, our astronomic time being only an approximation of. The summary of his meditations might be found in his "Mathematical Principles of Natural Philosophy": "I do not define time, space, place, and motion, as being well-known to all."

Not belonging to these "all", I want to examine the very necessity of the conventional definitions. Intrinsic to our mind (i.e., being a priory, as in Kant's works) ideas of "space" and "time" suggest only some freedom of motion. However in the Newtonian scheme, the space is already supplied with the three-dimensional Euclidean geometry, that is, it is a somehow defined set of elements – positions – that form the non-compact metric space with all the related properties. The time is not merely "past-now-future" but also a one-dimensional metric space with the countable base of open sets (neighborhoods), and its metrics is monotonous. Why all these?

Imagine a body placed into empty space. How can we tell between its being at rest and moving? The question is quite senseless provided nothing else is there. A reference frame is this "else" in the Newtonian mechanics. Only then can we define the trajectory of this body using readings on the reference frame axes. Still, this frame is only an auxiliary means in the problem. But why do we need to know this trajectory? This becomes meaningful only if some other bodies may come into contact with this one, and it is this contact that is in question of any real problem in mechanics and generally – in physics.

The purpose actually consists in predictions of the contacts, implying the further action to influence the reaching of this contact. Then, why do we need an intermediary like an external reference frame, rather than to directly consider only the motion of the bodies of interest in our problem? If the event of contact in question does occur, the coordinates of the bodies coincide at some time moment. Hence, the trajectories must (in the Newtonian mechanics) be written in numbers as time-functions of the coordinates taken from the reference frame. Only if times for different trajectories are appropriately coordinated, the predictions of contacts become possible. The accepted solution is one time for all trajectories in the problem, and the synchronized clocks are needed at each position in the reference frame.

All this rather complex measurement system is feasible, provided:

(i) Synchronizing signals connect all positions of the reference frame instantly. Believing that "for any fast motion a faster one might be found", an overcoming signal must always be used, so that observation of the body that could come into contact of interest would never have been lost.

(ii) Suitable clocks are to be made somehow. In daily life rough astronomical timing: years, months, days, hours, might be inappropriate. However, the design of mechanical clocks is based on the previously established principles of mechanics that are still under examination in our essay (One more circle!).

(iii) Identity of the clocks periods is perfect.

The second Newton's law describes some external influence on the trajectory – a force. The idea consists in integrating the series of free trajectories' segments to approximate the actual trajectory as altered from the free motion by this (smooth) force. The end points of each segment contact those of its neighbors. With the reference frame readings their lengths can be used to obtain the measure for integration. The transitions between the segments normalized to the related time intervals define the proportional to the force 'acceleration' as the measure for the transitions between the segments. Leaving aside the mathematical details of these approximations and their limits to the Calculus, I want to focus on the very measurement of a force in Newtonian mechanics. Indeed, where to find the vector of the force? Traditionally, some particular kind of forces is suggested for the problem of interest like the gradient of an external potential (as, e.g., in oscillations, gravity), friction, electromagnetic field etc. There is no general concept of force in the geometrical terms of the scheme itself. Provided the force is given in advance all over space-time, the whole trajectory can be found step by step. However, this approach cannot produce a genuine prediction as yet, being dependent on the knowledge of force up to the final state where no prediction is already interesting. In the Newtonian mechanics, inertia determining acceleration makes the scheme really predictable: Given the force, a sufficiently big mass of the body will send this force to the second order perturbation in the trajectory determining equation. It is just the demand of predictability that is responsible for second order terms in the equations to be sufficient: Force collected over the first order linear segment provides the next inter-segment transition, and no higher order terms are needed to determine them. So, the specification of only the initial free segment suffices to predict the final contact. This fact is not always understood, especially by mathematicians, believing in the known from experiment harmony of Nature. For instance, V. Arnold in his famous textbook "Mathematical Methods of Classical Mechanics" declared: "It is possible to fancy a world, in which for the determination of the future of a system one has to know in the initial moment also acceleration. Experiment shows that our world is not such."

However, any statement and result of experiment is formulated in terms of the already accepted theoretical models (Einstein: "In order to measure the velocity of light, the theoretical concept of velocity is necessary."). All these concepts originate in predictability. As a matter of fact, there is no harmony in our world, but the demand for predictability bounds us to develop a scheme ready for advising the person, looking obliviously around for the solution of his problem, to try first of all physics for the reaching of his wanted state.

The third Newton's law introduces the concept of interaction between bodies as a sole source of force, so providing some certainty to the second Newton's law. Then, an isolated from external influences collection of either or not interacting bodies taken as a whole must move freely according to the first Newtonian law. In particular, a solid body, considered as comprised of two parts separated with an infinitesimally thin gap, moves freely while, according to the second Newton's law, an additional force would be needed to keep each part moving free in spite of their reciprocal attraction. Hence we have to admit that the action of one part on the other is compensated by the opposite action.

3 Alternative numbering of motion

Newton considered velocities of bodies extendable in their values up to infinity, and then the using of located in advance clocks and rods became indispensible. Success in geometry tempted the using of the trajectory as the basic entity to start a theory with. On the contrary, the existence of the top-speed signal makes it possible to suggest a different numbering of motion. In so doing, we need no metric – no rods, no clocks, no material points, no reference frames. Our main concept is "contact", defined solely by its existence – "yes/no". The concept of body will be used just as a picturesque representation of contacts. It is the prediction of a contact using some auxiliary contacts – the Contact Problem (CP), that is the only issue of physics as a method to make universal predictions whenever relevant.

Attempts to define the space-time geometry with trajectories of limited velocities have been carried out in the middle of the past century [1-6]. In the interior of the light cone, trajectories were used to define neighborhoods generating the space-time topology as sets of points (events) such that any trajectory reaching a point of the neighborhood starting from outside passes also some other points of it, and there is some open interval in the order of the 1-dimensional continuum of this trajectory contained in this neighborhood (see Ref. 7 for details).

Consider two bodies A and B moving, each one along its (ordered) trajectory, toward their possible contact denoted (A,B). Let a set of auxiliary bodies be simultaneously emitted from A so that some of them reach B. Find the first of them to come into contact with B in the own B-order (One might imagine this first to put a mark on B, so that others meet B already marked.). Such a body will be taken for the top-speed signal, provided the emitted set is rich enough to cover all possible applications. A top speed must exist in the scheme for B not to be lost from observation upon its accelerations, so making predictions impossible. In so doing, we don't provide this top speed with a numerical value (no cm/sec, just topmost as defined!). Let further B emit instantly in response a similar set to reach A; it might be regarded as 'reflected' from B. This procedure being multiple repeated will be called the oscillation of the top-speed signal between A and B.

Our scheme of numbering motion consists solely in counting the numbers of these oscillations n_{AB}. Let us start this counting at some state of A. If (A,B) exists, the number of the oscillations is infinite, since were it finite some last oscillation before (A,B) will be there, in contradiction with the top-speed property of the signal, since either A or B would then reach (A,B) sooner. It is tempting to take the infinity of nAB for the prediction of the contact, but in the absence of (A,B) this number is still infinite though in the Newtonian scheme it would take infinite time; but we claim to use no measure for time, only the order.

In order to obtain the prediction, we can use an auxiliary body X with (A,X) known in advance and measure the ratio n_{AB}/n_{AC} for the triple (A,B,X), beginning at arbitrary point. (Both numbers being infinite, the ratio doesn't depend on this point.) The prediction of (A,B) follows from that of (A,B,X) provided such X can be found that this ratio is finite. Again, this is not a genuine prediction as yet, because we are counting the ratio up to the (A,B,X), and then nothing is left to predict. Hence, a scheme is to be developed to predict (A,B) already at the beginning of the oscillation numbers (ON) counting. Although we dispensed with all Newtonian intermediaries and turned to measure a motion solely by means of some auxiliary motions, we have yet to develop a scheme similar to the Newtonian to obtain genuine predictability.

For this to be possible, we ought now to consider suitable for our numbering scheme intersections of trajectories that allow for using the related concept of force. To this end, we define first the class Q of trajectories, the contacts between which are not too dense, so that with ON counting it be always possible to distinguish contacts however multiple. For instance, two trajectories, which in the Newtonian version have contacts only in all points with rational values of even one of coordinates, don't belong to Q. Hence, if trajectories from Q have two or more mutual contacts, ON counting, wherever started, might become infinite for only one of these. Only trajectories from Q are suitable for CP.

If the top-speed body signal S emitted from A at some of its point to contact B at some of its point, then no body emitted from A simultaneously with S can contact B in all points earlier than (S,B) in the B-order. So, we have now points in A and B that cannot be connected with trajectories unlike that in the Newtonian scheme. The set of all points, no pair of which can be so connected is called "spacelike hypersurface" W, and its elements will be called positions; therefore the trajectory of A, say, can contact W only at a single position. In particular, all top-speed signals connecting a point of A apart from W define some boundary in W: Only positions of W within this boundary can be connected with the part of A bounded by this point. An open in its order interval of A, crossing W at some of its points can be projected on W inside this boundary. This can be done using a series of mutually "parallel" trajectories (The notion of parallelism might be defined using a system of four ratios of ON's, and so defined parallel trajectories are not necessarily straight lines.) as follows. Take r points on A such that the finite ON's between neighboring pairs of parallel trajectories, connecting them to W, differ by only one oscillation. Increase r keeping this condition. In the limit r going to infinity we obtain a path of positions in W, which are in one-to-one correspondence with the set of r points in A trajectory to form (again being completed with irrational limits) the one-dimensional continuum. Unlike trajectories, paths might have self-intersections, though "rarefied" in accord with the trajectories they are projections of.

The whole W is an "envelope" for various combinations of possibly intersecting paths. If paths intersect, then the contact of their trajectories either exists or not. However, if paths don't intersect no contact can be there. It is only this purely topological property that is important for CP. W must have enough freedom to allow all the variety of combinations of passes. Since paths and their allowed combinations are one-dimensional, they might be topologically embedded in the 3-dimensional Euclidean space (Remember traffic interchanges. In general, a wide class of n-dimensional spaces, including our paths, might be so embedded in the Euclidean space of the dimension 2n+1, according to the Noebeling-Pontryagin theorem. Hence the geometry of space, taken in the Newtonian scheme as fallen from heaven, merely results from the union of all paths, and more dimensions for W would be redundant, because already some 3-dimensional subspace of it can include all cases for CP. Importantly, W cannot be considered as a sub-space of the 4-dimentional Lorentz spacetime, otherwise its meaningless topology with non-countable neighborhoods would be only1-dimensional in both Lebesgue and Poincare senses.

A top-speed signal cannot have more than one contact with any other trajectory in our scheme. Some other trajectories might have single contacts too, and these will be useful to define a force. Let us therefore select a special class of trajectories – the measurement X-kit with the following properties:

(i) Two trajectories from X either have no contacts or have only one;

(ii) Any point of a trajectory from Q has contacts with some trajectories from X.

(iii) Any two points of a trajectory from Q can be connected by a trajectory from X. Free trajectories of the first Newton's law are such, and just these properties of them, perhaps only locally, are actually needed in our scheme too.

In the second Newton's law acceleration is determined by force. Let us now inverse this law so as to determine force via acceleration, though not of the body of interest in the CP but of a body from the specially prepared auxiliary test P-kit with the same scheme of contacts as the X-kit, however comprised of bodies with some fixed constants to be specified for the particular kind of forces. Provided such standard constants exist over the whole Q, one is able to determine the acceleration of the body A that is of interest in CP comparing its acceleration at each point to that of the test body from the P-kit here, given the related constants of both. If the bodies participating in this comparison differ from each other only by the values of their constants, the trajectory of A can be defined, and therefore it is worthwhile to represent a force as the product of a constant and an entity defined by the ON counting – field. With the definition of our two kits, the said comparison might always be achieved with the counting of ON's and their ratios. The mentioned properties of the kits are specified just to allow for this comparison, so defining situations, in which we claim to make reliable predictions.

In the chain of links approximating a trajectory with a broken line, it is sufficient to specify only the first link. Then the force defining inter-link transitions (given the required constants) provide the prediction.

It remains now to define the required constants in terms of ON. We specify first a regular P-star, comprised of trajectories of some P's from the P-kit with the common contact, in which the ON ratios are distributed regularly:

(i) Each trajectory of P has the neighbors, that is, a number of trajectories, the ratios of the ON between P and any its neighbor to that between P and any other trajectory from the star exceeds 1; it follows that the ratios of the ON between P and any pair of its neighbors equals 1.

(ii) This feature is the same for all trajectories of the star.

In 3-dimensional Q these conditions can be exact only for a star with the configuration of a Platonic solid (If a star comprises great many trajectories, this inexactness might be ignored in the definition of a measure as the numbers of trajectories in subsets of the star; this is used, e.g., in the problems of field propagation, however not referred to further on in this essay restricted to mechanics.).

Consider a Platonic solid star with the bodies from the P-kit moving from its vertices toward the center solely under their interactions (Remember the third Newton law.). It is convenient to describe the gauge procedure for the constants in Newtonian terms (translation into the ON counting will be evident). These bodies are assumed to have some masses m and charges q. The completely identical bodies can reach the center only being mutually attracted as for gravity; otherwise some charge compensation is needed. Then only two of the Platonic solids might be relevant: the cube and the icosahedron. Indeed, in both it is possible to distribute opposite charges so as to obtain a regular star for bodies from P-kit.

The cube might be arranged out of two interwoven tetrahedrons – one with $+q$, another with $-q$; hence the star is neutral as a whole. All 8 initial velocities are radial and equal, and 8 equal initial radii are also the same for all bodies of the cube. All these bodies are being equally accelerated proportionally to q^2/m toward the center along rays, whatever radial dependence of their (isotropic) interaction force. We ascribe the cube star to electromagnetic (EM) interaction, the magnetic component of which is then equal 0 on the rays, and the electric field is purely radial.

Starting ON counting from the initial radii, we find their ratios for each ray with its neighbors to be 1 for any n. Reversing argument, the value 1 of these ratios can be taken as the criterion for the cube star to be perfect. After passing the star center the bodies decelerate to reach initial velocity at the same radii as the initial ones. Here some of them can be used, with an appropriate order of the vertices, to form the descendant star from this seed, adding more similar bodies. A triple of the neighboring seed star bodies completely determines all other members of the descending star with ON ratios counting. In the progress of this descending step by step in all directions, the charge and mass are transported over the whole network in Q, so determining the same pair of standard constants everywhere. Importantly, both m and q must be the same in the cube: Varying any of them in a part of cube, even keeping the value q^2/m unchanged, destroys the star symmetry. Hence, the network transports both standards unchanged.

In a more general case of CP, e.g., with an arbitrary external EM field, the source of which is not known in advance, unlike that in the Newtonian approach, the acceleration of charged bodies is proportional to the q/m rather than to q^2/m. However, the value of q/m is also determined by the cube star gauge, since both q and m are preserved upon the descent transportation. So, predictions based on ON counting are available in CP even beyond the Newtonian scheme.

The icosahedron regular star of oppositely charged bodies (also neutral as a whole) exists only if, in the Newtonian sense, the interaction force increases with radius. Whereas the cube is a sub-star of the full dodecahedron, the icosahedron stands alone; hence its charge and mass have nothing in common with EM q and m. With the distance increasing of its force, allowing for confinement and asymptotic freedom, the icosahedron star symmetry might be suggested to explain the Dark Cold Matter and the Dark Energy in cosmology.

4 Postscript

The origin of the "Laws of Nature" for any method of numbering motion as well as of the concept of motion itself results merely from the very problem statement by the person-user to find, whenever possible, a universally predictable course of action. To this end, physics suggests CP. Nature has no harmony of its own; only living creatures are looking for reliable schemes to make predictions. In particular, it is clear now why quantum mechanics had not developed its own variables instead of classical position and momentum. However modified, these variables still present information in terms required by the user.

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