Propagation of a Particle in Discrete Time

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In the concept of discrete time, we can guess the causal delay. A new analysis of causal delays in the dynamics can provide views of two different worlds: type 1 and type 2. In the case of a free particle, the evolution operator for each of them was obtained and analyzed. As a result, type 1 particle could be interpreted as ordinary matter that satisfies existing relativistic quantum mechanics. Type 2 particle is outside the quantum mechanics category, but has some interesting physical properties. Type 2 particle acts on gravity in the same way as ordinary matter, and does not interact with the U(1) gauge field, and considering its energy density value, it can be interpreted as dark matter.

1 Introduction

The dynamical system aims to find dynamic variables that change over time, which is the process of solving the equations of motion. The structure of the equation of motion combines the amount of change of the dynamic variables with time and the cause of the change.

As an example, let's take a look at Newton's laws of motion. Newton expressed his second law as follows:

> *Change of motion is proportional to impressed motive force and is in the same direction as the impressed force.*

The equation of motion is as follows.

$$\lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t} = \vec{F}(t) \ .$$

Applying the cause-effect category to Newton's law of motion mentioned above, force is the cause and momentum changes are the effects.

However, the point to note here is the time difference between the moment t when the cause force is applied and $t + \Delta t$ which is the moment when the resultant momentum change appears. Naturally, in continuous space and time, this time difference is infinitely small, so the cause and effect are "simultaneous". Let's call this simultaneity *infinitesimally different simultaneity*. This infinitesimally different simultaneity is assumed in all dynamic systems based on continuous space and time: Newtonian mechanics, Lagrangian mechanics, Hamiltonian mechanics, Quantum mechanics, etc.

By the way, this infinitesimally different simultaneity is two different points, unlike *true simultaneity* which is identical. Because if they are the same, then at any moment an object has to have both momentum before the cause and momentum after the cause. The distinction between two points in infinitesimally different simultaneity in continuous space and time is meaningless, but in discrete time, there is a minimum value Δt_d for time change and two points for cause and effect, resulting in a delay of time Δt_d between cause and effect.

The delay between cause and effect will of course affect the description of the dynamics, which requires an evolution operator for a particle in discrete time. There are two types of results in this process, one that is consistent with existing relativistic quantum mechanics and another that is entirely new.

2 Definitions

2.1 Cause-effect vectors

Considering the causal delay, we cannot define the "real state" at one moment, as in quantum mechanics, and define the "real state" within the minimum time Δt_d. The existing quantum mechanical state with 4-momentum p_μ at a point x^α in space-time can be called $\phi_p(x^\alpha)$ which is caused by $x^\alpha - \Delta x^\alpha$ or $x^\alpha + \Delta x^\alpha$ due to the causal delay. Where Δx^α is a timelike 4-vector, meaning cause-effect delation in space-time. The time component of Δx^α is an amount representing the cause-effect time delation Δt_d and the spatial component represents the distance the object moved during the time delay.*

Therefore, the definition of the "real state" in discrete time must be made by combining coordinate values with $\phi_p(x^\alpha)$. So there are two definitions, past-future cause-effect vector and future-past cause-effect vector. Where $\phi_p(x^\alpha)$ is tentatively scalar.

past-future cause-effect vector : $x^\mu \phi_p(x^\alpha + \Delta x^\alpha)$,

future-past cause-effect vector : $(x^\mu + \Delta x^\mu)\, \phi_p(x^\alpha)$.

2.2 Difference of cause-effect vectors

Since there are two states between Δx^α as discussed above, by combining them, the state change can be of two types:

$$\text{type 1} : (x^\mu + \Delta x^\mu)\, \phi_p(x^\alpha) - x^\mu \phi_{p'}(x^\alpha + \Delta x^\alpha) \ . \qquad (1)$$

$$\text{type 2} : x^\mu \phi_{p'}(x^\alpha + \Delta x'^\alpha) - (x^\mu - \Delta x^\mu)\, \phi_p(x^\alpha) \ . \qquad (2)$$

*In this paper, unlike time, distance in space does not assume its minimum value. The discreteness of space is a controversial topic and has nothing to do with the content of this paper.

The two types are shown schematically in Fig. 1.* As shown in Fig. 1, type 1 is the difference between future-past cause-effect vector and past-future cause-effect vector, and type 2 consists only of the difference between past-future cause-effect vectors.

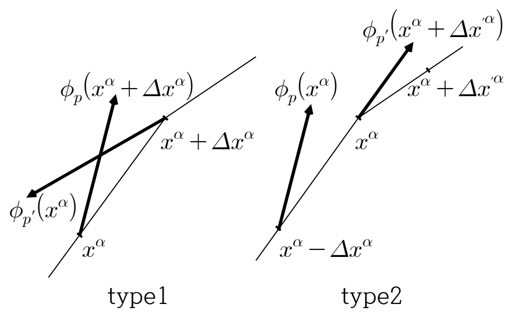

Fig. 1: Schematics of type 1 and type 2

3 Calculations of difference of cause-effect vectors for a free particle

Assumption 1 : In type 1, the state value $\phi_p(x^\alpha)$ at point x^α has the same magnitude of contribution at $x^\alpha - \Delta x^\alpha$ and $x^\alpha + \Delta x^\alpha$.

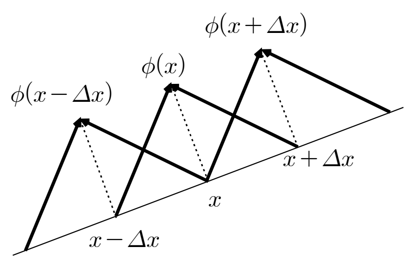

Fig. 2: Contributions to each ϕ

In Fig. 2, $\phi(x)$ is a mixture of contributions from $x - \Delta x$ and $x + \Delta x$. The same applies to the other ϕ's. This can be written in the following way. p is omitted because it is the same.

$$\phi(x) = \phi_{x-\Delta x}(x) + \phi_{x+\Delta x}(x) \ . \tag{3}$$

This contribution in space-time is shown in Fig. 3.

As shown in the Fig. 3, the two contributions to $\phi(x)$ at x^μ will act in opposite directions on the tangent of the dotted world line. Thus both contributions will be offset. The same is true for vectors. That is, scalars and vectors cannot describe type 1.

*In discrete time, the trajectory in space-time cannot be a solid line, only a jump from point to point. The solid trajectory in Fig. 1 and 2 is just for readability.

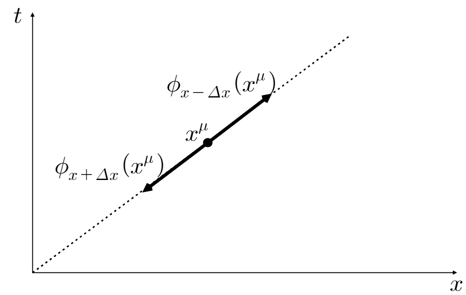

Fig. 3: two contributions to $\phi(x)$

What about the spinor? In Fig. 3, the two spinors are also the same magnitude and in opposite directions, but the sum is not zero. Because in spinor space the two spinors are orthogonal. Two orthogonal spinors correspond to $\binom{1}{0}$ and $\binom{0}{1}$, respectively, which correspond to spin $\frac{1}{2}$. Thus, only spin $\frac{1}{2}$ spinors can describe type 1.

Now, if the spinor is constant, the difference of cause-effect vectors for a 2-component spinor $\Psi_a(x)$ is defined as follows.

$$\text{type 1 : } (x^\mu + \Delta x^\mu)\,\Psi_a(x) - x^\mu \Psi_a(x + \Delta x) \ . \tag{4}$$

In the case of a free particle, the difference of cause-effect vectors for type 2 is also shown.

$$\text{type 2 : } x^\mu \phi(x + \Delta x) - (x^\mu - \Delta x^\mu)\,\phi(x) \ . \tag{5}$$

Assumption 2 : Ψ and ϕ are analytic functions.

But in reality it is discontinuous and it is difficult to figure this out. This assumption approximates discontinuous Ψ and ϕ as C^∞ functions, which means looking at the dynamical point of view that we are familiar with.

3.1 Type 1

Let's express the spinor function $\Psi_a(x)$ as a spinor part and a scalar part depending on the coordinates as follows.

$$\Psi_a(x) = u_a \phi(x) \ . \tag{6}$$

Then, (4) is as follows.

$$u_a \left\{(x^\mu + \Delta x^\mu)\,\phi(x) - x^\mu \phi(x + \Delta x)\right\} \ . \tag{7}$$

So we only need to calculate the part for scalar.

$$\begin{aligned}
&(x^\mu + \Delta x^\mu)\,\phi(x^\alpha) - x^\mu \phi(x^\alpha + \Delta x^\alpha) \\
&= (x^\mu + \Delta x^\mu)\,\phi(x^\alpha) - x^\mu \sum_{n=0}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n \phi(x^\alpha) \\
&= \Delta x^\lambda \left\{\delta^\mu_\lambda \phi(x^\alpha) - x^\mu \frac{\partial \phi(x^\alpha)}{\partial x^\lambda}\right\} - \\
&\quad - x^\mu \sum_{n=2}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n \phi(x^\alpha) \ .
\end{aligned}$$

For $n \geq 2$

$$x^\mu \left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n = \left[x^\mu, \left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n\right]_{\text{commutation}} .$$

Thus

$$\begin{aligned}
&(x^\mu + \Delta x^\mu)\,\phi(x^\alpha) - x^\mu \phi(x^\alpha + \Delta x^\alpha) \\
&= -\left[x^\mu, \Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right]\phi(x) - \\
&\quad - \left[x^\mu, \sum_{n=2}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n\right]\phi(x) \\
&= -\left[x^\mu, \sum_{n=1}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n\right]\phi(x) \\
&= -\left[x^\mu, \exp\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right) - 1\right]\phi(x) \\
&= -\left[x^\mu, \exp\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)\right]\phi(x) \,.
\end{aligned}$$

For the progress of the calculation, we define the 4-momentum operator P_λ, and commutation relation of x^μ and P_λ as follows.

$$\begin{aligned}
P_\lambda &\equiv i\hbar \frac{\partial}{\partial x^\lambda} \\
[x^\mu, P_\lambda] &\equiv -i\hbar \delta^\mu_\lambda
\end{aligned} \tag{9}$$

and metric $\eta_{\alpha\beta} = \text{diag}\begin{bmatrix}1 & -1 & -1 & -1\end{bmatrix}$.

Using the following (10), the final result is as shown in (11).

$$\begin{aligned}
[x_i, F(P_i)] &= i\hbar \frac{\mathrm{d}F}{\mathrm{d}P_i} \\
[x_0, F(P_0)] &= -i\hbar \frac{\mathrm{d}F}{\mathrm{d}P_0} \,.
\end{aligned} \tag{10}$$

$$\begin{aligned}
&(x^\mu + \Delta x^\mu)\,\phi(x^\alpha) - x^\mu \phi(x^\alpha + \Delta x^\alpha) \\
&= \Delta x^\mu \exp\left(-\frac{i}{\hbar}\,\Delta x^\alpha P_\alpha\right)\phi(x) \,.
\end{aligned} \tag{11}$$

Therefore, the equation for spinor function $\Psi_a(x)$ is

$$\begin{aligned}
&(x^\mu + \Delta x^\mu)\,\Psi_a(x) - x^\mu \Psi_a(x + \Delta x) \\
&= \Delta x^\mu \exp\left(-\frac{i}{\hbar}\,\Delta x^\alpha P_\alpha\right)\Psi_a(x) \,.
\end{aligned} \tag{12}$$

3.2 Type 2

After a similar calculation process as in type 1, the equation that corresponds to (12) is (16).

$$\begin{aligned}
&x^\mu \phi(x + \Delta x) - (x^\mu - \Delta x^\mu)\,\phi(x) \\
&= x^\mu \sum_{n=0}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n \phi(x) - (x^\mu - \Delta x^\mu)\,\phi(x) \\
&= \Delta x^\alpha \left(x^\mu \frac{\partial}{\partial x^\alpha} + \delta^\mu_\alpha\right)\phi(x) + x^\mu \sum_{n=2}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n \phi(x) \,.
\end{aligned}$$

For $n \geq 2$

$$x^\mu \left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n = \left\{x^\mu, \left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n\right\}_{\text{anticommutation}} .$$

$$\begin{aligned}
&x^\mu \phi(x + \Delta x) - (x^\mu - \Delta x^\mu)\,\phi(x) \\
&= \left\{x^\mu, \Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right\}\phi(x) + \left\{x^\mu, \sum_{n=2}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n\right\}\phi(x) \\
&= \left\{x^\mu, \sum_{n=1}^{\infty} \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n\right\}\phi(x) \\
&= \left\{x^\mu, \exp\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right) - 1\right\}\phi(x) \,.
\end{aligned} \tag{13}$$

Note that unlike type 1, anticommutation occurs. Therefore, for calculation, we need to define the anticommutation relation of 4-vector x and P as below.

$$\begin{aligned}
P_\lambda &\equiv i\hbar \frac{\partial}{\partial x^\lambda} \\
\{x^\mu, P_\lambda\} &\equiv i\hbar \delta^\mu_\lambda \,.
\end{aligned} \tag{14}$$

And using the following (15), the final equation corresponding to (12) of type 1 is the following (16).

$$\begin{aligned}
\{x_i, G(P_i)\} &= -i\hbar \frac{\mathrm{d}G}{\mathrm{d}P_i} \\
\{x_0, G(P_0)\} &= i\hbar \frac{\mathrm{d}G}{\mathrm{d}P_0} \,.
\end{aligned} \tag{15}$$

$$\begin{aligned}
&x^\mu \phi(x + \Delta x) - (x^\mu - \Delta x^\mu)\,\phi(x) \\
&= \left(\Delta x^\mu \exp\left(-\frac{i}{\hbar}\Delta x^\alpha P_\alpha\right) - 2x^\mu\right)\phi(x) \,.
\end{aligned} \tag{16}$$

To understand the meaning of the right sides of (12) and (16) for type 1 and type 2, we first briefly review the time evolution operator in quantum mechanics in the next chapter. In a similar manner, in space-time, the right side of (12) will be defined as the evolution operator of the type 1 particle and the right side of (16) will be defined as the evolution operator of the type 2 particle.

4 Evolution operator

In quantum mechanics, the time evolution operator U from state $|\,\alpha, t_0\,\rangle$ to $|\,\alpha, t_0; t\,\rangle$ is defined as follows.

$$|\,\alpha, t_0; t\,\rangle = U(t, t_0)\,|\,\alpha, t_0\,\rangle \,.$$

Determining this time evolution operator is equivalent to determining the equation of motion for the state of the system. If the time evolution operator is as follows, this operator satisfies the Schrodinger equation.

$$\begin{aligned}
&U(t, t_0) = \exp\left(-\frac{iH(t - t_0)}{\hbar}\right) \\
&i\hbar \frac{\partial U(t, t_0)}{\partial t} = HU(t, t_0) \\
&i\hbar \frac{\partial}{\partial t} U(t, t_0)\,|\,\alpha, t_0\,\rangle = HU(t, t_0)\,|\,\alpha, t_0\,\rangle \,.
\end{aligned}$$

That is, the Schrodinger equation for the state is established as below.

$$i\hbar \frac{\partial}{\partial t} \mid \alpha, t_0; t \rangle = H \mid \alpha, t_0; t \rangle .$$

Now let's discuss type 1 and type 2. Type 1 and type 2 discussed here are free particles, so we can apply the concept of evolution operator in quantum mechanics.

4.1 Type 1

$$\begin{aligned}
&(x^\mu + \Delta x^\mu)\, \Psi_a(x) - x^\mu \Psi_a(x+\Delta x) \\
&= \Delta x^\mu \exp\left(-\frac{i}{\hbar}\Delta x^\alpha \hat{P}_\alpha\right)\Psi_a(x) \\
&\equiv \Delta x^\mu U(\Delta x)\,\Psi_a(x)\ .
\end{aligned} \tag{17}$$

Suppose $U(\Delta x)$ in (17) is an evolution operator by Δx in space-time. Then

$$\begin{aligned}
&\Psi_a^p(\Delta x) \equiv U(\Delta x)\,\Psi_a^p(0) \\
&= \exp\left(-\frac{i}{\hbar}\Delta x \cdot \hat{P}\right)\Psi_a^p(0) = \exp\left(-\frac{i}{\hbar}\Delta x \cdot\ p\right)\Psi_a^p(0)\ .
\end{aligned} \tag{18}$$

Successive evolution by $n\Delta x^\alpha = x^\alpha$ can be expressed as below.

$$\begin{aligned}
&\exp\left(-\frac{i}{\hbar}\Delta x\cdot\hat{P}\right)\cdots\cdot\exp\left(-\frac{i}{\hbar}\Delta x\cdot\hat{P}\right)\Psi_a^p(0) \\
&= \exp\left(-\frac{i}{\hbar}n\Delta x\cdot p\right)\Psi_a^p(0) = \exp\left(-\frac{i}{\hbar}x\cdot p\right)\Psi_a^p(0) \\
&= \exp\left(-\frac{i}{\hbar}x\cdot\hat{P}\right)\Psi_a^p(0) = U(x)\,\Psi_a^p(0)\ .
\end{aligned}$$

Therefore, $U(x)$ satisfies the following equation.

$$\Psi_a^p(x) = U(x)\,\Psi_a^p(0)\ . \tag{19}$$

If we apply the Klein-Gordon operator to (19) and use p as constant, we get the following result ($\hbar = 1$).

$$\begin{aligned}
&\left(\partial_\mu\partial^\mu + m^2\right)\Psi_a^p(x) \\
&= \left(\partial_\mu\partial^\mu + m^2\right)e^{-ix\cdot\hat{P}}\Psi_a^p(0) \\
&= \left(\partial_\mu\partial^\mu + m^2\right)e^{-ix\cdot p}\Psi_a^p(0) \\
&= \left(-p_\mu p^\mu + m^2\right)e^{-ix\cdot p}\Psi_a^p(0) \\
&= 0\,.
\end{aligned} \tag{20}$$

As you can see from (20), $\Psi_a^p(x)$ is the solution of the Klein-Gordon equation. And, as discussed above, $\Psi_a^p(x)$ is spin $\frac{1}{2}$, so $\Psi_a^p(x)$ can be said to be a component of a spinor that satisfies the Dirac equation.

In summary, for a free particle, type 1 can be interpreted as a conventional ordinary matter that satisfies the Dirac equation, and $U(x)$ can be interpreted as an evolution operator. It is also worth noting that type 1 particles, although their beginnings are unusual, are in agreement with existing relativistic quantum mechanics, indicating some of the validity of the causal delay.

4.2 Type 2

$$\begin{aligned}
&x^\mu\phi(x+\Delta x) - (x^\mu - \Delta x^\mu)\,\phi(x) \\
&= \left(\Delta x^\mu exp\left(-\frac{i}{\hbar}\Delta x^\alpha P_\alpha\right) - 2x^\mu\right)\phi(x) \\
&\equiv \Delta x^\mu V\phi(x)\ .
\end{aligned} \tag{21}$$

We have dicussed that U of type 1 can be interpreted as an evolution operator. Based on that, we will define V as an evolution operator of type 2. But unlike U, V is not a unitary operator, i.e. type 2 particles are broken in unitarity. Nevertheless, type 2 particles have very interesting physical meanings.

5 Properties of type 2 particle

5.1 $x^\mu \gg \Delta x^\mu$

The evolution operator at large x is

$$\Delta x^\mu V \simeq -2x^\mu \tag{22}$$

Since V is a linear function of x in (22), the equation that V must satisfy is the second order differential equation, i.e. $\partial_\alpha\partial_\beta V = 0$. Thus, the equation of motion that x must satisfy can be written in covariant form as

$$\frac{d^2 x^\mu}{d\tau^2} = 0 \tag{23}$$

where τ is the proper time.

Eq. (23) is the classical relativistic equation of motion for a free particle, not a wave equation. This means that in large x there is only motion as a particle and no quantum waves.

More discussion is needed about the above. If we take $\Delta x \to 0$ limit on both sides in (16), it is as follows.

$$\begin{aligned}
&x^\mu\left(\phi(x) + \Delta x^\alpha\frac{\partial\phi}{\partial x^\alpha}\right) - (x^\mu - \Delta x^\mu)\,\phi(x) \\
&= \Delta x^\mu\left\{\phi(x) - \frac{i}{\hbar}\Delta x^\alpha p_\alpha\phi(x)\right\} - 2x^\mu\phi(x) \\
&\Delta x^\alpha\frac{\partial\phi(x)}{\partial x^\alpha} = -2\phi(x)\ .
\end{aligned}$$

$$\therefore\ \phi(x) \propto \exp\left(-2\frac{\Delta x\cdot x}{\Delta x\cdot\Delta x}\right). \tag{24}$$

As shown in (24), the particle position is very localized. However, this is the position value in the state where the momentum is determined. In other words, type 2 particle can be determined at the same time the position and momentum, which means that there is no quantum wave phenomenon in the type 2 particle. Although quantum waves do not exist, it has a physical meaning because it satisfies the classical equation of motion.

Eq. (23) holds for an inertial frame in flat spacetime. If a curved spacetime manifold is locally flat at an arbitrary point P, (23) always holds at P because $\frac{\partial}{\partial x^\gamma}g_{\alpha\beta}(P) = 0$. This means that in locally flat manifolds, type 2 particle undergo free-falling motion with a straight geodesic. That is, type 2 particle is affected by gravity in the same way as ordinary matter.

5.2 $x^\mu \simeq \Delta x^\mu$

The evolution operator in this case is as follows.

$$V \simeq \exp\left(-\frac{i}{\hbar} x \cdot \hat{P}\right) - 2 \,. \tag{25}$$

The first term in (25) is the operator giving the Klein-Gordon equation. The second term, as discussed in 5.1, means acceleration, which is related to mass. Thus, (25) can be seen as an operator that gives an equation that modifies the mass part of the Klein-Gordon equation. If the modified Klein-Gordon equation is set as shown in (26) below, f is obtained as follows ($\hbar = 1$).

$$\left(\partial_\mu \partial^\mu + m^2 f\right) \phi_p(x) = 0 \,. \tag{26}$$

where $\phi_p(x) = V(x)\,\phi_p(0)$.

$$\begin{gathered} \left(\partial_\mu \partial^\mu + m^2 f\right)\left(e^{-ix\cdot\hat{P}} - 2\right) \phi_p(0) = 0 \,. \\ \therefore\ f(x) = \frac{e^{-ix\cdot p}}{e^{-ix\cdot p} - 2} \,. \end{gathered} \tag{27}$$

In the modified Klein-Gordon equation, the mass term is a complex number.

We will now discuss the internal symmetry of type 2 particles.

The equation that satisfies $\phi(x)$ and $\phi^*(x)$ in (26) is as follows.

$$\begin{gathered} \left(\partial_\mu \partial^\mu + m^2 f(x)\right) \phi(x) = 0 \\ \left(\partial_\mu \partial^\mu + m^2 f^*(x)\right) \phi^*(x) = 0 \,. \end{gathered} \tag{28}$$

As can be seen from (28), the equations satisfying $\phi(x)$ and $\phi^*(x)$ are different. This means that the type 2 particles do not have antiparticles and, as will be seen later, do not have internal symmetry. To show that the type 2 particles do not have internal symmetry, the Lagrangian density must be determined. However, defining the Lagrangian density implies that type 2 is assumed to be a field only locally, although this is not the case for large x. In addition, the Lagrangian density should be a locally holomorphic complex Lagrangian.

By the way, the Lagrangian density of a normal complex scalar field cannot produce (28). Therefore, some process is required.

First, changing the expression (28) using $f^*(x) = f(-x)$ is as follows.

$$\left(\partial_\mu \partial^\mu + m^2 f(-x)\right) \phi^*(x) = 0 \,.$$

And $x \to -x$ gives

$$\left(\partial_\mu \partial^\mu + m^2 f(x)\right) \phi^*(-x) = 0 \,. \tag{29}$$

In (28) and (29), it can be seen that $\phi(x)$ and $\phi^*(-x)$ satisfy the same equation. Thus Lagrangian density can be written as

$$\mathcal{L} = \partial_\mu \phi^*(-x)\, \partial^\mu \phi(x) - m^2 f(x)\, \phi^*(-x)\, \phi(x) \,. \tag{30}$$

Now consider the following gauge transformations.

$$\begin{gathered} \phi(x) \to e^{-iq\theta(x)} \phi(x) \,, \ \ \phi^*(-x) \to e^{iq\theta(-x)} \phi^*(-x) \,. \\ \frac{\delta\phi(x)}{\delta\theta(x)} = -iq\phi(x) \,, \ \ \frac{\delta\phi^*(-x)}{\delta\theta(x)} = iq\phi^*(-x)\, \frac{\delta\theta(-x)}{\delta\theta(x)} \,. \end{gathered} \tag{31}$$

According to Noether's theorem, if the action is invariant under gauge transformations, there is a vanishing divergence current, whose value is

$$\begin{aligned} J^\mu &= -\left[\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi(x)\right)} \frac{\delta\phi(x)}{\delta\theta(x)} + \frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi^*(-x)\right)} \frac{\delta\phi^*(-x)}{\delta\theta(x)}\right] \\ &= iq\left[\phi(x)\, \partial^\mu \phi^*(-x) - \phi^*(-x)\, \partial^\mu \phi(x)\, \frac{\delta\theta(-x)}{\delta\theta(x)}\right] . \end{aligned} \tag{32}$$

However, the divergence of current in (32) is not zero. And this means that there is no conserved charge.

Therefore, we can say that Lagrangian density in (30) has no internal symmetry. In other words, type 2 particles cannot construct covariant derivatives that satisfy gauge invariance, which means that type 2 particles do not interact with the U(1) gauge field.

In addition, since type 2 particles lack an intenal symmetry, it means that it is a kind of scalar without components, so SU(2) and SU(3) gauge symmetry cannot be defined. Accordingly, type 2 particles do not have weak interactions and strong interactions as well as electromagnetic interactions, but is connected only by gravity.

5.3 Mass and energy density

Let's first discuss the mass of type 2 particle.

In the modified Klein-Gordon equation, mass is distributed in space-time like a wave, and its distribution is determined by f. Therefore, to find the mass of type 2 particle, we need to find the integral value for the space. Let $t = 0, \vec{p} = (p, 0, 0)$ be for simplicity. Then f is

$$f(x) = \frac{e^{ipx}}{e^{ipx} - 2} \,. \tag{33}$$

In (33), f diverges at $x \to \pm\infty$. This is because the expression (33) holds for $x^\mu \simeq \Delta x^\mu$. Therefore, to find the integral, we need to define a function value at $x \to \pm\infty$. As discussed in section 5.1, type 2 particles do not have a wave function for large x. Consequently, we can set the boundary condition $f \to 0$ at $x \to \pm\infty$. In order for f to converge at $x \to \pm\infty$, we need to modify f in (33). By introducing damping factor ϵ, modified f is presented as below.

$$f_m = \frac{e^{i(p+i\epsilon)|x|}}{e^{i(p+i\epsilon)x} - 2} \,. \tag{34}$$

Therefore, the mass M of type 2 particle can be defined as (35). And m is the mass of ordinary matter that satisfies Klein-Gordon equation.

$$M^2 \equiv m^2 \left| \int_{-\infty}^{\infty} \frac{e^{i(p+i\epsilon)|x|}}{e^{i(p+i\epsilon)x} - 2} dx \right| . \tag{35}$$

In order to calculate the integral value of (35), the following integral of a complex variable must be obtained.

$$\oint dz \frac{e^{i(p+i\epsilon)|z|}}{e^{i(p+i\epsilon)z} - 2} . \tag{36}$$

The poles and residues are

$$\text{simple pole} \quad z_0 = -\frac{(\epsilon + ip)}{\epsilon^2 + p^2} \ln 2 .$$

residue at z_0

$$\begin{aligned} a_{-1} &= \frac{e^{i(p+i\epsilon)|z|}}{e^{i(p+i\epsilon)z} - 2} \cdot \left(e^{i(p+i\epsilon)z} - 2 \right) |_{z=z_0} \\ &= 2^{i \frac{(p+i\epsilon)}{\sqrt{p^2+\epsilon^2}}} \\ &= 2^i \quad (\text{for } \epsilon \to 0) . \end{aligned}$$

The contour of integration is shown in Fig. 4.

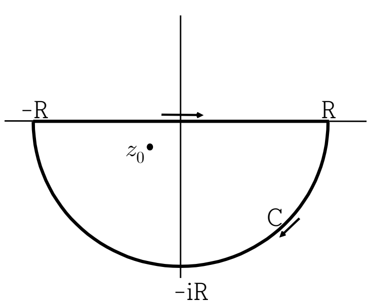

Fig. 4: Contour of integration

$$\begin{aligned} &\oint dz \frac{e^{i(p+i\epsilon)|z|}}{e^{i(p+i\epsilon)z} - 2} \\ &= \lim_{R\to\infty} \int_{-R}^{R} dx \frac{e^{i(p+i\epsilon)|x|}}{e^{i(p+i\epsilon)x} - 2} + \int_C dz \frac{e^{i(p+i\epsilon)|z|}}{e^{i(p+i\epsilon)z} - 2} \\ &= 2\pi i a_{-1} = 2\pi i \cdot 2^i . \end{aligned} \tag{37}$$

Since the second integral term in (37) is 0 as $R \to \infty$, the mass value to be obtained is

$$\frac{M^2}{m^2} = \left| 2\pi i \cdot 2^i \right| = 2\pi . \tag{38}$$

Let's discuss the energy density. First, the energy density of type 1, that is, ordinary matter, is as follows in the case of a complex scalar field.

$$\mathcal{L} = \partial_\mu \phi^* (x) \, \partial^\mu \phi (x) - m^2 \phi^* (x) \, \phi (x)$$

$$T^{\mu\lambda} = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_i} \partial^\lambda \phi_i - \eta^{\mu\lambda} \mathcal{L} = \partial^\mu \phi^* \partial^\lambda \phi + \partial^\mu \phi \partial^\lambda \phi^* - \eta^{\mu\lambda} \mathcal{L} .$$

Accordingly, the energy density in the case of a free particle is as follows.

$$T^{00} = \left(\vec{\mathrm{p}}^2 + m^2 \right) \phi^* \phi .$$

Ignoring the kinetic part:

$$T^{00}_{\text{type 1}} \simeq m^2 \left| \phi_1 \right|^2 . \tag{39}$$

Lagrangian density and energy momentum tensor of type 2 are as follows.

$$\begin{aligned} \mathcal{L} &= \partial_\mu \phi^* (-x) \, \partial^\mu \phi (x) - m^2 f (x) \, \phi^* (-x) \, \phi (x) \\ T^{\mu\lambda} &= \partial^\mu \phi^* (-x) \, \partial^\lambda \phi (x) + \partial^\mu \phi (x) \, \partial^\lambda \phi^* (-x) - \eta^{\mu\lambda} \mathcal{L} . \end{aligned} \tag{40}$$

However, as discussed earlier, the Lagrangian density of type 2 is a complex number, so the energy momentum tensor of the above formula is also a complex number. Therefore, the above energy momentum tensor cannot be applied to the physical system as it is.

This issue is intended to find meaning through the following discussion. As discussed earlier, type 2 has no internal symmetry, so there is no short distance interaction. That is, only long distance interaction (gravity) is possible. However, at far distances, there is no wave property, but only particle properties, so type 2 has only meaning as particles in the long distance interaction. Acting as a particle means that it participates in gravity as a particle having a mass M of the type 2 obtained above. In this case, the behavior of the particles as mass M is equivalent to ordinary matter. Therefore, the energy density of type 2 can be treated as the energy density of the scalar field of mass M. Accordingly, the same process as the energy density of type 1 discussed above is as follows.

$$T^{00}_{\text{type 2}} \simeq M^2 \left| \phi_2 \right|^2 . \tag{41}$$

Consequently, the energy density ratio of type 1 and type 2 particles is as follows.

$$\frac{T^{00}_{\text{type 2}}}{T^{00}_{\text{type 1}}} \simeq \frac{M^2 \left| \phi_2 \right|^2}{m^2 \left| \phi_1 \right|^2} . \tag{42}$$

One thing to note here is that the mass m of ordinary matter compared in the above formula is the mass of the particle as a free particle.

In (42), $\left| \phi_2 \right|^2 / \left| \phi_1 \right|^2$ is the ratio of mass-independent amplitudes, so we can make them equal. Accordingly, the following results can be obtained.

$$\frac{T^{00}_{\text{type 2}}}{T^{00}_{\text{type 1}}} \simeq \frac{M^2}{m^2} = 2\pi = \frac{86.3\%}{13.7\%} . \tag{43}$$

Since type 1 and type 2 have the same opportunity for generation in their origin, the number density of the two will be the same. Therefore, the ratio of the above equation is the ratio of the energy density of the total amount of type 1 and type 2 in the universe. The value is within the range of the energy density ratios of dark matter and ordinary matter that are currently estimated.

6 Conclusions

The interpretation of the dynamical system with a new concept of causal delay, which originated from the discrete concept of time, gave us a perspective on two different worlds. For a free particle, type 1 particle can be interpreted as ordinary matter that satisfies existing relativistic quantum mechanics. This type 1 particle can only have spin $\frac{1}{2}$, which can explain why the spin of all fermions observed is $\frac{1}{2}$.

Type 2 particle is a matter of a whole new perspective. This particle does not follow the existing laws of quantum mechanics. Type 2 is only a classical particle that satisfies the theory of relativity at a long distance, and has a property as a kind of field that does not have gauge symmetry at a short distance. So, these type 2 particles act on gravity in the same way as ordinary matter, do not interact with light, and considering their energy density value, it can be interpreted as dark matter.

Type 2 particles do not have any gauge interactions. And there is no antiparticle, including itself, so no annihilation occurs. Therefore, direct or indirect detection based on them is not possible, only indirect verification through gravity. However, given the local nature of type 2, it is not a point-like particle, so self-interaction through collision seems to be possible.

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