Physics of Irrational Numbers

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In systems of coupled periodic processes, lasting frequency ratios can cause significant physical effects, which depend on the type of real numbers the ratios are approximating. Rational frequency ratios can cause parametric resonance and amplification, while approaching irrational frequency ratios can avoid them. In this paper we discuss physical effects that can be caused by frequency ratios approximating some irrational algebraic and transcendental numbers. We illustrate this approach on some features of the solar system which are still unexplained.

Introduction

In this paper, we introduce an approach that bases on the physical interpretation of certain statements of the number theory. In modern theoretical physics, numerical ratios usually remain outside the realm of theoretical interest. In this work we try to elucidate the physical meaning of numerical ratios and to show their theoretical and practical importance for resolving some fundamental problems of physics.

One of the unsolved fundamental problems in physics [1] is the stability of systems of a large number of coupled periodic processes, for instance, the stability of planetary systems. If numerous bodies are considered to be gravitationally bound to one another, perturbation models predict long-term highly unstable states [2] that contradict the physical reality of the solar system and thousands of exoplanetary systems.

In our previous publications we have applied our numeric-physical approach to the analysis of the orbital and rotational periods of the planets, planetoids and moons of the solar system and thousands of exoplanets [3] with the conclusion that the avoidance of orbital and rotational parametric resonances by approximation of transcendental ratios can be viewed as a basic forming factor of planetary systems [4].

Another unsolved fundamental problem is the imperishability of motion and interaction, and the inexhaustibility of energy. This question seems to be out of the realms of modern physics. Indeed, until now, all the sources of energy we are currently using – from electricity to radioactivity – were discovered by chance. This fact and the incapacity of inventing new energy sources evidences the lack of comprehention. For instance, the research of the predicted thermonuclear fusion has been going on for 60 years without success [5, 6].

Likewise, the nature of gravitational energy is still a mystery [7]. For instance, what is the propelling force of the orbital motion? Naturally, there is no propelling of orbital motion, the planets are in perpetual free fall. However, the orbital velocity of a planet is very high, 30 kilometers per second in the case of the Earth. The impulse of a planet is therefore enormous and sweeps away everything that gets in its trajectory. Where does this kinetic energy come from? Perhaps, this question seems naive to the physicist who is ready to answer immediately: Besides the primordial kinetic energy of the protoplanetary disk, the potential energy of the gravity field of the star is the source of the kinetic energy of planetary motion. However, this answer only readdresses the question. Then what is the source of gravitational energy? Is it the alleged ability of a mass to curve space-time? Then what causes this ability?

Obviously, the concept of mass is not complete since the numerical values of particle masses still remain a mystery. Where do the observed masses of elementary particles come from? This is the biggest, and oldest, unresolved enigma in fundamental particle physics. There is the widespread, but erroneous, belief that the Higgs boson resolves the origin of particle masses. This is not the case. It merely replaces one set of unknown parameters (particle rest energies) with an equally unknown set of parameters (coupling constants to the Higgs field), so nothing is gained in the fundamental understanding of masses [8].

Is there a hidden inexhaustible source of energy in the universe? Then why can energy not be generated or consumed, but only converted?

The earliest constants of motion discovered were momentum and kinetic energy, which were proposed in the 17^{th} century by René Descartes and Gottfried Leibniz on the basis of collision experiments, and later refined by Euler, Lagrange, d'Alembert and Hamilton. In theoretical physics, Noether's first theorem connects the conservation of energy with the homogeneity of time, supposing that the laws of physics do not change over time. Noether's theorem states that conservation laws apply in a physical system with conservative forces. A conservative force is a force with the property that the total work done in moving a particle between two points is independent of the path taken. Equivalently, if a particle travels in a closed loop, the total work done by a conservative force is zero. In short, a conservative force is a force that conserves energy. Hence, Noether's theorem leads to circular reasoning. It does not explain the cause of energy conservation [9]. Perhaps, no physical principle can explain the origin of energy, because every physical process presupposes the existence of another physical process that serves as its energy source. This non ending chain of energy converters suggests that the imperishability of motion and interaction, and the inexhaustibility of energy must have a non-physical cause.

Our numeric-physical approach leads us to the conclusion that motion and interaction, including energy as well as other constants of motion are caused by attractors of numeric fields. We illustrate this conclusion on some features of the solar system which are still unexplained.

Theoretical Approach

The starting point of our approach is frequency as obligatory characteristic of a periodic process. As the result of a measurement is always a *ratio* of physical quantities, one can measure only *ratios* of frequencies. This ratio is always a real number. Being a real value, a frequency ratio can approximate an integer, rational, irrational algebraic or transcendental number. In [10] we have shown that the difference between rational, irrational algebraic and transcendental numbers is not only a mathematical task, but it is also an essential aspect of stability in systems of bound periodic processes. For instance, integer frequency ratios, in particular fractions of small integers, make possible parametric resonance that can destabilize such a system [11, 12]. This is why asteroids cannot maintain orbits that are unstable because of their resonance with Jupiter [13]. These orbits form the Kirkwood gaps that are areas in the asteroid belt where asteroids are absent.

According to this idea, irrational ratios should not cause destabilizing resonance interactions, because irrational numbers cannot be represented as a ratio of integers. However, algebraic irrational numbers, being real roots of algebraic equations, can be converted to rational numbers by multiplication. For example, the algebraic irrational number $\sqrt{2} = 1.41421\ldots$ cannot become a frequency scaling factor in real systems of coupled periodic processes, because $\sqrt{2} \cdot \sqrt{2} = 2$ creates the conditions for the occurrence of parametric resonance. Thus, only transcendental ratios can prevent parametric resonance, because they cannot be converted to rational or integer numbers by multiplication. Actually, it is transcendental numbers, that define the preferred frequency ratios which allow to avoid destabilizing resonance [14]. In this way, transcendental frequency ratios sustain the lasting stability of coupled periodic processes. With reference to the evolution of a planetary system and its stability, we may therefore expect that the ratio of any two orbital periods should finally approximate a transcendental number [15].

However, the issue is to clarify the type of number a measured ratio corresponds to. Because of the finite resolution of any measurement, there is no possibility to know it for sure. The obtained value is always an approximation and therefore, it is very important to know the amount of its uncertainty.

It is remarkable that approximation interconnects all types of real numbers – rational, irrational algebraic and transcendental. In 1950, Aleksandr Khinchin [16] made a very important discovery: He could demonstrate that simple continued fractions deliver biunique representations of all real numbers, rational and irrational. Whereas infinite continued fractions represent irrational numbers, finite continued fractions represent always rational numbers. In this way, any irrational number can be approximated by finite continued fractions, which are the convergents and deliver always its nearest and quickest rational approximation.

It is notable that the best rational approximation of an irrational number by a finite continued fraction is not a task of computation, but only an act of termination of the continued fraction recursion. For example, the golden ratio $\phi = (\sqrt{5}+1)/2 = 1.618\ldots$ has a biunique representation as simple continued fraction that contains only the number 1:

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}$$

As the continued fraction of ϕ is periodic, it meets a quadratic equation evidencing that ϕ is algebraic:

$$\phi = 1 + \frac{1}{\phi} \qquad \phi^2 - \phi - 1 = 0$$

In order to save space, in the following we use angle brackets to write down continued fractions, for example the golden ratio $\phi = \langle 1; 1, 1, \ldots \rangle$. So long as the sequence of denominators is considered as infinite, this continued fraction represents the irrational number ϕ. If the continued fraction will be truncated, the sequence of denominators will be finite and we get a convergent that is always the nearest rational approximation of the irrational number ϕ.

In the case of ϕ, the approximation process is very slow because of the small denominators. Only the 10^{th} approximation gives the correct third decimal of ϕ. In fact, the denominators in the continued fraction of ϕ are the smallest possible and consequently, the approximation speed is the lowest possible. The golden ratio ϕ is therefore treated as the 'most irrational' number in the sense that a good approximation of ϕ by rational numbers cannot be given with small quotients. On the contrary, transcendental numbers can be approximated exceptionally well by rational numbers, because their continued fractions contain large denominators and can be truncated with minimum loss of precision. For instance, the simple continued fraction of Archimedes' number $\pi = 3.1415927\ldots = \langle 3; 7, 15, 1, 292, \ldots \rangle$ delivers the following sequence of rational approximations:

$\langle 3 \rangle = 3$
$\langle 3; 7 \rangle = 22/7 = 3.\overline{142857}$
$\langle 3; 7, 15 \rangle = 3.1\overline{4150943396226}$
$\langle 3; 7, 15, 1 \rangle = 3.1415929\ldots$

Already the 2nd approximation delivers the first two decimals correctly. Therefore, 22/7 is a widely used Diophantine approximation of π. The 4th approximation shows already six correct decimals. This special arithmetic property of continued fractions [17] of transcendental numbers has the consequence that transcendental numbers are distributed near by rational numbers of small quotients or close to integers, like $e^3 = 20.08\ldots$ or $\pi^3 = 31.006\ldots$. This can create the impression that complex systems like the solar system provide ratios of physical quantities that approximate rational numbers. More likely, they approximate transcendental numbers [4], which are located close to rational numbers.

Naturally, a continued fraction of π or any other real transcendental number cannot be periodic, otherwise it would meet an algebraic equation. For example, the continued fractions of the algebraic irrationals $\sqrt{2} = \langle 1; 2, 2, 2, \ldots \rangle$ and $\sqrt{3} = \langle 1; 1, 2, 1, 2, \ldots \rangle$ are periodic. In contrast to them, a generalized continued fraction of Euler's number contains all natural numbers in sequence as numerators and denominators and therefore, it cannot be periodic:

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{4 + \ldots}}}}$$

The following generalized continued fraction [18] of π contains all natural numbers factorizing the numerators:

$$\pi = 2 + \cfrac{2}{1 + \cfrac{1 \cdot 2}{1 + \cfrac{2 \cdot 3}{1 + \cfrac{3 \cdot 4}{1 + \ldots}}}}$$

These continued fractions do not only evidence that π and e are not algebraic, but make comprehensible the increase of the approximation speed with every next convergent. In addition, it becomes clear that Archimedes' number π can be approximated faster than Euler's number e.

Among all transcendental numbers, Euler's number $e = 2.71828\ldots$ is unique, because its real power function e^x coincides with its own derivatives. In the consequence, Euler's number allows avoiding parametric resonance between any coupled periodic processes including their derivatives.

Because of this unique property of Euler's number, we expect that periodic processes in real systems prefer frequency ratios close to Euler's number and its roots. The natural logarithms of those frequency ratios are therefore close to integer $0, \pm 1, \pm 2, \ldots$ or rational $\pm 1/2, \pm 1/3, \pm 1/4, \ldots$ values. For rational exponents, the natural exponential function is always transcendental [19]. Since every rational number has a biunique representation as a simple finite continued fraction, we can represent the logarithms of the frequency ratios we are looking for as finite continued fractions:

$$\ln(\omega_A / \omega_B) = \mathcal{F} = \langle n_0; n_1, n_2, \ldots, n_k \rangle \tag{1}$$

ω_A and ω_B are the angular frequencies of two bound periodic processes A and B avoiding parametric resonance. We use angle brackets for continued fractions. All denominators $n_1, n_2, \ldots, n_k$ of a continued fraction including the free link n_0 are integer numbers. All numerators equal 1. The length of a continued fraction is given by the number k of layers.

The canonical form (all numerators equal 1) does not limit our conclusions, because any continued fraction with partial numerators different from 1 can be transformed into a canonical continued fraction using the Euler equivalent transformation [20]. Therefore, finite canonical continued fractions represent all rational numbers in the sense that there is no rational number that cannot be represented as a finite canonical continued fraction. This universality of canonical continued fractions evidences that the distribution of rational logarithms (1) is fractal. As it is an inherent feature of the number continuum, we call it the *Fundamental Fractal* [14].

The first layer of this fractal is given by the truncated after n_1 continued fractions:

$$\langle n_0; n_1 \rangle = n_0 + \frac{1}{n_1}$$

The denominators n_1 follow the sequence of integer numbers ± 1, ± 2, ± 3 etc. The second layer is given by the truncated after n_2 continued fractions:

$$\langle n_0; n_1, n_2 \rangle = n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2}}$$

Figure 1 shows the first and the second layer in comparison. As we can see, reciprocal integers $\pm 1/2, \pm 1/3, \pm 1/4, \ldots$ are the attractor points of the fractal. In these points, the distribution density of rational logarithms (1) reaches a local maximum. Integers $0, \pm 1, \pm 2, \ldots$ define the main attractors having the widest ranges. Half logarithms $\pm 1/2$ form smaller attractor ranges, third logarithms $\pm 1/3$ form the next smaller ranges and so forth. Increasing the length of the continued fraction (1), the distribution density of the transcendental frequency ratios ω_A / ω_B is increasing as well. Nevertheless, their distribution is not homogeneous, but fractal. Applying continued fractions and truncating them, we can represent the logarithms $\ln(\omega_A / \omega_B)$ as rational numbers $\langle n_0; n_1, n_2, \ldots, n_k \rangle$ and make visible their fractal distribution.

The linear projection $\mathcal{E} = \exp(\mathcal{F})$ of the fundamental fractal (fig. 1) is a fractal scalar field of transcendental attractors that we call the *Euler field* [3]. Figure 2 (central part) shows the 2D-projection of its first layer. The Euler field is topologically 3-dimensional, a fractal set of embedded

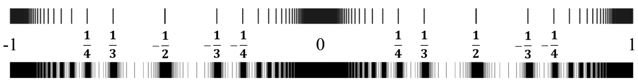

Fig. 1: The distribution of rational logarithms for $k = 1$ (above) and for $k = 2$ (below) in the range $-1 \leqslant \mathcal{F} \leqslant 1$.

spheric equipotential surfaces. The potential difference defines a gradient directed to the center of the field that causes a central force of attraction creating the effect of a field source. Because of the fractal logarithmic hyperbolic metric of the field, also every equipotential surface is an attractor. The logarithmic scalar potential difference $\Delta\mathcal{F}$ of sequent equipotential surfaces equals the difference of sequent continued fractions (1) on a given layer:

$$\Delta\mathcal{F} = \langle n_0; n_1, \ldots, n_k \rangle - \langle n_0; n_1, \ldots, n_k + 1 \rangle$$

Main equipotential surfaces at $k = 0$ correspond with integer logarithms; equipotential surfaces at deeper layers $k > 0$ correspond with rational logarithms.

The Euler field is of pure arithmetic origin, and there is no particular physical mechanism required as field source. Hence, we postulate the universality of the Euler field that should affect any type of physical interaction, regardless of its complexity. Corresponding with (1), the Euler field generates a fractal set of transcendental frequency ratios $\omega_A/\omega_B = \mathcal{E}$ which allow to avoid destabilizing parametric resonance and in this way, provide the lasting stability of periodic processes bound in systems regardless of their complexity. This conclusion we have exemplified [21] in particle physics, astrophysics, geophysics, biophysics and engineering.

In several publications we have shown that the Euler field determines the orbital periods of thousands of exoplanets and large bodies in the solar system [3] as well as their gravitational parameters [4]. Astrophysical and geophysical cycles [22] as well as periodic biophysical processes [10] obey the Euler field. Finally, the Euler field determines the proton-to-electron ratio and the W/Z-to-electron ratio as well as the temperature 2.725 K of the cosmic microwave background radiation [14]. All these findings suggest that the cosmological significance of the Euler field is that of a universal stabilizer.

The radii of the equipotential surfaces of the Euler field $\mathcal{E} = e^{\mathcal{F}}$ are integer and rational powers of Euler's number. However, not only Euler's number $e = 2.71828\ldots$ defines a fractal scalar field of its integer and rational powers, but in general, every prime, irrational and transcendental number does it. While the fundamental fractal (fig. 1) is always the same distribution of rational logarithms, the structure of the corresponding fundamental field changes with the logarithmic base. Here it is important to notice that no fundamental field can be transformed in another by scaling (stretching), because $\log_a(x) - \log_b(x)$ is a nonlinear function of x. In this way, every prime, irrational or transcendental number generates a unique fundamental field of its own integer and rational powers that causes physical effects which are typical for that number. For instance, the golden ratio $\phi = \langle 1; 1, 1, \ldots \rangle$ makes difficult its rational approximation, since its continued fraction does not contain large denominators. Hence, the fundamental field of its integer and rational powers should be a perfect inhibitor of resonance amplification. We propose to name this field after Hippasus of Metapontum who was an ancient Greek philosopher and early follower of Pythagoras, and is widely credited with the discovery of the existence of irrational numbers, and the first proof of the irrationality of the golden ratio. Figure 2 (left part) shows the 2D-projection of the first layer of the Hippasus field $\mathcal{H} = \phi^{\mathcal{F}}$.

Although the golden ratio is irrational, it is a Pisot number, so its powers are getting closer and closer to whole numbers. This is why the Hippasus field can inhibit resonance within small frequency ranges only. Euler's number is not a Pisot number, so that the Euler field permits coupled periodic processes to avoid parametric resonance also over very large frequency ranges. Since the natural logarithm of the golden ratio is close to 1/2, small powers of the golden ratio can approximate main equipotential surfaces of the Euler field. For example, $\phi^2 = 2.618\ldots$ can serve as approximation of $e = 2.718\ldots$ Within small frequency ranges, this circumstance makes the Hippasus field a fast and simplified local approximation of the Euler field. In fact, as the continued fraction of the golden ratio contains only the number 1, approximations of the golden ratio can be achieved faster than approximations of Euler's number, since every extension of its continued fraction requires counting and additional computing. Therefore, systems of coupled periodic processes follow the Hippasus field within small frequency ranges only. For example, several authors [23, 24] have suggested that the Venus-to-Earth orbital period ratio 0.615 approximates the golden ratio $1/\phi = 0.618\ldots$ preventing Earth and Venus from parametric orbital resonance. However, the Hippasus field cannot prevent the whole solar system from orbital resonance. For instance, the Pluto-to-Venus orbital period ratio does not obey a power of the golden ratio, but approximates the 6^{th} power of Euler's number [10]. The 6^{th} power of Euler's number is in the range of the 12^{th} power of the golden ratio that approximates a whole number and hence cannot serve as a scaling factor that prevents parametric resonance.

Obviously, in systems with many coupled periodic processes, the Hippasus field can produce two opposing effects: over small frequency ranges, the Hippasus field can inhibit parametric resonance, but over large frequency ranges, it provides the long-period appearance of resonance amplification.

Furthermore in this paper, we introduce the Archimedes field $\mathcal{A} = \pi^{\mathcal{F}}$. Figure 2 (right part) shows the 2D-projection of its first layer. The radii of the equipotential surfaces of the Archimedes field are integer and rational powers of π.

According to our numeric physical approach, we interpret the fact that circumference / radius = π in the way that the transcendence of π makes possible circular motion. The transcendence of the circumference avoids interruptions and makes impossible to define the start or endpoint of motion. Furthermore, Archimedes number π makes possible eternal oscillation. This is why it is impossible to *completely* stop oscillations, for example, the thermal oscillations of atoms. According to our approach, the origin of the zero point energy phenomenon lies in the transcendence of π.

Proven by Theodor Schneider [25] in 1937, the perimeter of an ellipse is transcendental. Elliptical or circular motion is the only way to move with acceleration without propulsion. The absence of propulsion makes this motion eternal. In this way, the transcendence of π makes possible eternal accelerated motion. Hence, Archimedes' number appears to be a universal source of kinetic energy and promoter of orbital and rotational motion.

In the framework of our approach, gravity is a physical effect caused by numeric attractors [3]. They cause mass accretion forming a celestial body and determine its movement in space and time. In this way, planets, stars, planetary systems and galaxies are materializations of numeric attractors. These attractors exist long before a star or planet is formed. In order to reach an attractor, the accelerated displacement of matter causes the force conventionally interpreted as gravity. Numeric attractors are primary; mass accretion is secondary. In this way, gravitation is not caused by the body mass, and it is not a physical property of a celestial body at all. We suppose that fundamental numeric attractors cause all types of physical interaction.

As well, the appearance of a field source is only a scaling effect. A field is not created by a charge, but the charge is a scaling effect of the field. The gradient of the field is the force of attraction that indicates the location of the energy source. The attractor is the energy source. Matter falls down to the attractor because in this way it gains energy. This is why the core of a planet is hot. On the contrary, in the assumption that mass is the source of gravity, and in accordance with Newton's shell theorem, the Preliminary Reference Earth Model [26] affirms the *decrease* of the gravity acceleration with the depth. However, this hypothesis is still under discussion. In 1981, Stacey and Holding [27, 28] reported anomalous measures (larger values than expected) of the gravity acceleration in deep mines and boreholes.

According to our approach, the acceleration of free fall should *increase* with the vicinity to the field singularity, but follow the logarithmically hyperbolic fractal metric of the fundamental numeric field. In [29] we have shown that the Euler field reproduces the 3D profile of the Earth's interior confirmed by seismic exploration. As well, the stratification layers in planetary atmospheres follow the Euler field [30].

Are there attractors of the Euler field that coincide with attractors of the Archimedes field? Since $e = 2.71828\ldots$ and $\pi = 3.14159\ldots$ are transcendental, there are no rational powers of these numbers that can produce identical results. Therefore, in general, Archimedes-attractors are different from Euler-attractors. However, some of them are so close to each other that they form common attractors. It is not difficult to compute the exponents of two transcendental numbers that define a common attractor. The ratio of their logarithms is a fractal dimension that equals $D = \ln \pi = 1.144729\ldots$ Representing D as continued fraction $\langle 1; 7, -11, \ldots \rangle$, we immediately find 8/7 as the first approximation. Consequently, multiples of 8/7 define pairs of Euler-attractors of stability and Archimedes-attractors of motion that are very close to each other. For example, this is valid for $\mathcal{E}\langle 56 \rangle$ and $\mathcal{A}\langle 49 \rangle$. We will study this and other examples in the paragraph *Exemplary Applications*. Naturally, our description of possible physical effects caused by the fields $\mathcal{A}, \mathcal{E}, \mathcal{H}$ does not claim to be complete.

Exemplary Applications

Let us start with an application of the Euler field that demonstrates its ability of avoiding parametric resonance over extremely large scale-differences. For instance, Venus' distance from Sun approximates the main equipotential surface $\mathcal{E}_e\langle 54 \rangle$ of the Euler field of the *electron* that equals the 54^{th} power of Euler's number multiplied by the Compton wavelength of the electron λ_e. The aphelion 0.728213 AU = $1.08939 \cdot 10^{11}$ m delivers the upper approximation:

$$\ln\left(\frac{A_O(Venus)}{\lambda_e}\right) = \ln\left(\frac{1.08939 \cdot 10^{11}\ \text{m}}{3.86159 \cdot 10^{-13}\ \text{m}}\right) = 54.00$$

The perihelion 0.718440 AU = $1.07477 \cdot 10^{11}$ m delivers the lower approximation:

$$\ln\left(\frac{P_O(Venus)}{\lambda_e}\right) = \ln\left(\frac{1.07477 \cdot 10^{11}\ \text{m}}{3.86159 \cdot 10^{-13}\ \text{m}}\right) = 53.98$$

This means that Venus' orbit derives from the Euler field of the electron. In other words, Venus' orbit is of subatomic origin. This is not a random coincidence. Jupiter's distance from Sun approximates the main equipotential surface $\mathcal{E}_e\langle 56 \rangle$ of the same electron Euler field. The aphelion 5.45492 AU = $8.160444 \cdot 10^{11}$ m delivers the upper approximation:

$$\ln\left(\frac{A_O(Jupiter)}{\lambda_e}\right) = 56.01$$

The perihelion 4.95029 AU = $7.405528 \cdot 10^{11}$ m delivers the lower approximation:

$$\ln\left(\frac{P_O(Jupiter)}{\lambda_e}\right) = 55.91$$

As well, Jupiter's orbital period 4332.59 days derives from the Euler field of the electron. In fact, it equals the 66^{th} power of Euler's number multiplied by the oscillation period of the electron ($\tau_e = \lambda_e / c = 1.28809 \cdot 10^{-21}$ s is the angular oscillation period of the electron):

$$\ln\left(\frac{T_O(Jupiter)}{2\pi \cdot \tau_e}\right) = \ln\left(\frac{4332.59 \cdot 86400\ \text{s}}{2\pi \cdot 1.28809 \cdot 10^{-21}\ \text{s}}\right) = 66.00$$

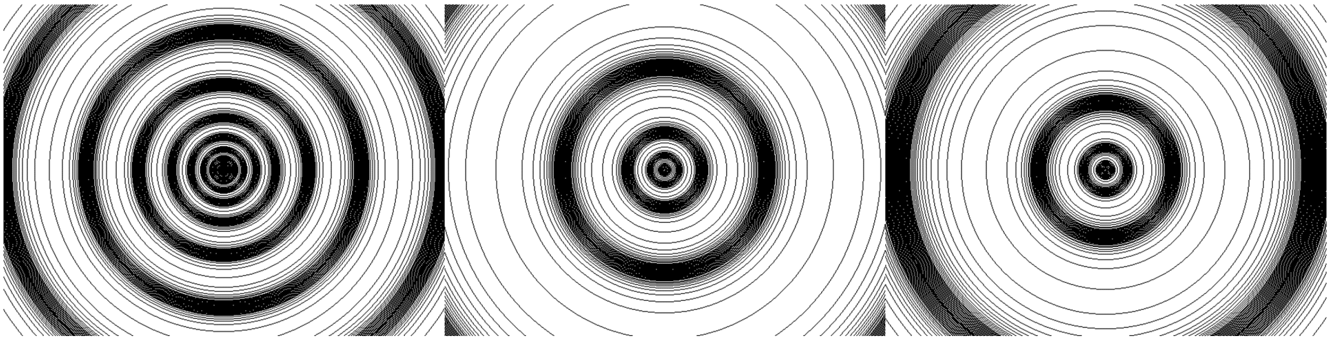

Fig. 2: The image shows the 2D-projection of the first layer ($k = 1$) of equipotential surfaces of the Hippasos Field $\mathcal{H} = \phi^{\mathcal{F}}$ (left), the Euler Field $\mathcal{E} = e^{\mathcal{F}}$ (center), and the Archimedes Field $\mathcal{A} = \pi^{\mathcal{F}}$ (right) of the Fundamental Fractal $\mathcal{F}$. The fields are shown to the same scale.

The same is valid for the orbital period 686.98 days (1.88 years) of the planet Mars that equals the 66^{th} power of Euler's number multiplied by the *angular* oscillation period of the electron:

$$\ln\left(\frac{T_{\mathrm{O}}(Mars)}{\tau_e}\right) = \ln\left(\frac{686.98 \cdot 86400\,\mathrm{s}}{1.28809 \cdot 10^{-21}\,\mathrm{s}}\right) = 66.00$$

Consequently, the ratio of the orbital periods of Jupiter and Mars equals 2π:

$$T_{\mathrm{O}}(Jupiter) = 2\pi \cdot T_{\mathrm{O}}(Mars)$$

This transcendental ratio allows Mars to avoid parametric orbital resonance with Jupiter and evidences that Jupiter and Mars are not planets of different systems, but bound together in the same conservative system (the solar system).

Also the orbital period 224.701 days of Venus derives from the Euler field of the electron, and it is stabilized by the main attractor $\mathcal{E}_e\langle 63\rangle$:

$$\ln\left(\frac{T_{\mathrm{O}}(Venus)}{2\pi \cdot \tau_e}\right) = 63.00$$

The complete (polar) rotational period $T_{\mathrm{R}}(Sun) = 34$ days of the Sun approximates the same attractor:

$$\ln\left(\frac{T_{\mathrm{R}}(Sun)}{\tau_e}\right) = 63.00$$

Consequently, the scaling factor 2π connects the orbital period of Venus with the rotational period of the Sun:

$$T_{\mathrm{O}}(Venus) = 2\pi \cdot T_{\mathrm{R}}(Sun)$$

Needless to say that these numeric relations cannot be derived from Kepler's laws or Newton's law of gravitation. Fig. 3 shows how Archimedes' number bonds together rotational and orbital periods. The scale symmetry of this connection not only reveals the Sun as the engine of planetary motion, but also the special role of Mercury. The connection of its rotation with the orbital motion of the Earth is surprising and encourages further investigation.

In general, orbital periods are stabilized by the Euler field of the electron, and rotational periods by the Euler field of the proton. For instance, the rotational periods of Earth and Mars derive from the angular oscillation period $\tau_p = \lambda_p/c$ of the proton ($\lambda_p = 2.10309 \cdot 10^{-16}$ m is the Compton wavelenght of the proton). They approximate the same attractor $\mathcal{E}_p\langle 67\rangle$. Mars' sidereal rotational period 24.62278 hours delivers the upper approximation:

$$\ln\left(\frac{T_{\mathrm{R}}(Mars)}{\tau_p}\right) = \ln\left(\frac{24.62278 \cdot 3600\,\mathrm{s}}{7.01515 \cdot 10^{-25}\,\mathrm{s}}\right) = 67.01$$

Earth' sidereal rotational period 23.93447 hours delivers the lower approximation:

$$\ln\left(\frac{T_{\mathrm{R}}(Earth)}{\tau_p}\right) = \ln\left(\frac{23.93447 \cdot 3600\,\mathrm{s}}{7.01515 \cdot 10^{-25}\,\mathrm{s}}\right) = 66.98$$

It is notable that the proton-to-electron ratio itself approximates the 7^{th} power of Euler's number and its square root:

$$\ln\left(\frac{\lambda_e}{\lambda_p}\right) = \ln\left(\frac{3.86159 \cdot 10^{-13}\,\mathrm{m}}{2.10309 \cdot 10^{-16}\,\mathrm{m}}\right) \simeq 7 + \frac{1}{2} = \mathcal{E}\langle 7; 2\rangle$$

In the consequence of this potential difference of the proton relative to the electron, the scaling factor $\sqrt{e} = 1.64872\ldots$ connects Euler field attractors of proton stability with similar attractors of electron stability in alternating sequence. In [4] we have applied Khinchine's [16] continued fraction method of approximation to the proton-to-electron ratio.

As we mentioned in the paragraph *Theoretical Approach*, multiples of 8/7 define pairs of Euler-attractors of stability and Archimedes-attractors of motion and energy that are very close to each other. For example, this is valid for $\mathcal{E}_e\langle 56\rangle$ and $\mathcal{A}_e\langle 49\rangle$, because $56/49 = 8/7$. This coincidence underlines the significance of the attractor $\mathcal{E}_e\langle 56\rangle$ that determines the orbit of the largest planet in the Solar system. If we apply the exponent 49 to Euler's number, we discover that $\mathcal{E}_e\langle 49\rangle$ corresponds with the radius of the Sun. In this way, the coincidence of $\mathcal{E}_e\langle 56\rangle$ with $\mathcal{A}_e\langle 49\rangle$ identifies the Sun as energy source and Jupiter as main orbital body of the Solar system.

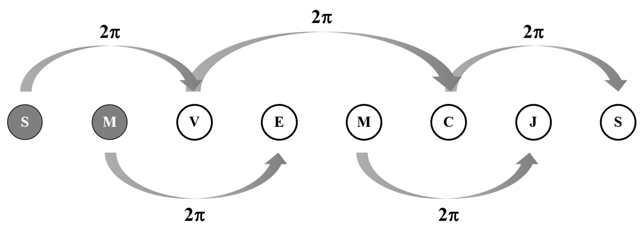

Fig. 3: From left to right: the *rotational* periods of the Sun (S) and Mercury (M), and the *orbital* periods of Venus (V), Earth (E), Mars (M), Ceres (C), Jupiter (J), and Saturn (S), coupled by the scaling factor 2π of the Archimedes field.

Interestingly, it is not the radius of the photosphere that coincides with the equipotential surface $\mathcal{E}_e\langle 49\rangle$, but the radius of the corona. It is noticeable that no complete theory yet exists to account for the extremely high temperature of the corona that reaches up to 20 million Kelvin. Despite great advances in observations and modelling, the problem of solar and stellar heating still remains one of the most challenging problems of space physics [31]. According to our approach, this heating could be a physical effect caused by numeric attractors of the Archimedes field.

Conclusion

According to our numeric-physical approach, numeric fields like $\mathcal{A}, \mathcal{E}$ are primary. Through their physical effects, they not only determine the frequency ratios of elementary particles, but also the setting of orbital and rotational periods in planetary systems. Modern theoretical physics is oriented towards equations, even if they cannot be solved. The language of equations is based on conservation rules, which, however, describe the behavior of model processes under certain ideal conditions of equilibrium. Nevertheless, the search for an equation describing the observed process is often considered a priority task of theoretical research. In this case, as a rule, numerical ratios are considered random. We consider this work to contribute to the idea that great unification in physics cannot be achieved as long as numerical ratios remain outside the realm of theoretical interest.

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